UNIT-1 AMPLITUDE MODULATION

Amplitude modulation - Generation and Detection of AM wave - Spectra - DSBSC, Hilbert Transform, Pre-envelope and Complex envelope - SSB and VSB - Comparison - Super heterodyne Receiver.

Introduction:-

* Communication: It is the process of conveying or transferring information (or) message from one point to another.

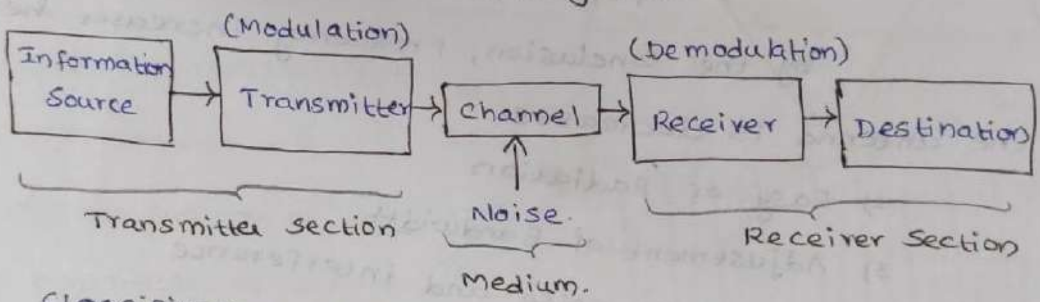
(or)

It simply link between transmitter and Receiver.

Example:- Telephony, Telegraphy, Radio broadcasting, mobile communication.

* Electromagnetic spectrum and spectrum allocation:- Total span of frequencies and corresponding wavelength used in communication system.

* Basic Element of communication system:-



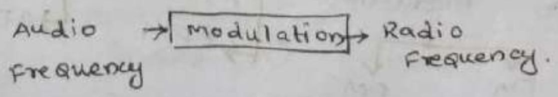
Classification of communication system depends on Channel (or) transmission medium is,

(i) Line communication (or) wired ; eg: coaxial, Twisted Fiber optic Cable

(ii) Radio (or) Wireless communication. eg: Mobile communication

* **MODULATION**:-

Modulation is the process of varying the characteristics parameter of amplitude, Frequency or Phase of the carrier signal according to its instantaneous value of message signal (or) modulating signal.



Because, of long distance Communication; signals are classified as,

* Base Band signal: [Without need of modulation (ie) short distance)
* Band pass (or) Pass band signal: [With modulation, long distance communication]

* Modes of Channel Operation:-

(i) Simplex → One way communication; eg: TV broadcast, Radio

(ii) Half Duplex → Two way communication not at a same time
eg: Walky talkie

(iii) Full Duplex → Two way communication at a same time.
eg: Mobile, Telephony n/w.

* NEED FOR MODULATION:-

1) Height of the transmitting and receiving antenna is reduced.

Practically; $h = \lambda/4$ for efficient transmission, $[\lambda = c/f]$

For 30Hz; $h = 2500$ km $c = 3 \times 10^8$

$f = 3$KHz; $h = 25$Km

$f = 3$MHz; $h = 25$m.

By the conclusion, Frequency increases height of the antenna is decreases.

2) Easy of Radiation
3) Adjustment of Bandwidth
4) To reduce noise and interference
5) Multiplexing
6) Narrow Banding.

TYPES OF COMMUNICATION

- Analog
  - Continuous wave (CW)
    - AM Amplitude modulation
    - Angle modulation
      - FM
      - PM
  - pulse modulation
    - PAM
    - PWM
    - PPM
- Digital
  - Carrier is pulse
    - PCM
    - DM
    - ADM
  - Carrier is Sinusoidal
    - ASK
    - FSK
    - QPSK
    - PSK
    - QAM

AMPLITUDE MODULATION [AM]

* **DEFINITION:-** Amplitude modulation is the process by which amplitude of the carrier signal is varied accordance with the instantaneous value of the modulating signal (message), but frequency and phase of the carrier wave remains constant.

* Classification of AM.

AM

AM with carrier (or) DSB-FC (Double side band Full carrier)	Double side band Suppressed carrier (DSB-SC)	Single side band suppressed carrier (SSB-SC)	Vestigial side band (VSB)	Independent side band suppressed carrier (ISB/SC)

Not in syllabus

(1) AM with carrier (or) DSB-FC

(or) AM ENVELOPE :-

* Mathematical Representation of AM (or) AM Envelope waveform:-

Let the modulating signal is represented as,

$V_m(t) = V_m \sin\omega_m t$ — ①

Carrier signal is represented as,

$V_c(t) = V_c \sin\omega_c t$ — ②

According to definition of AM, the amplitude of carrier signal is changed after modulation.

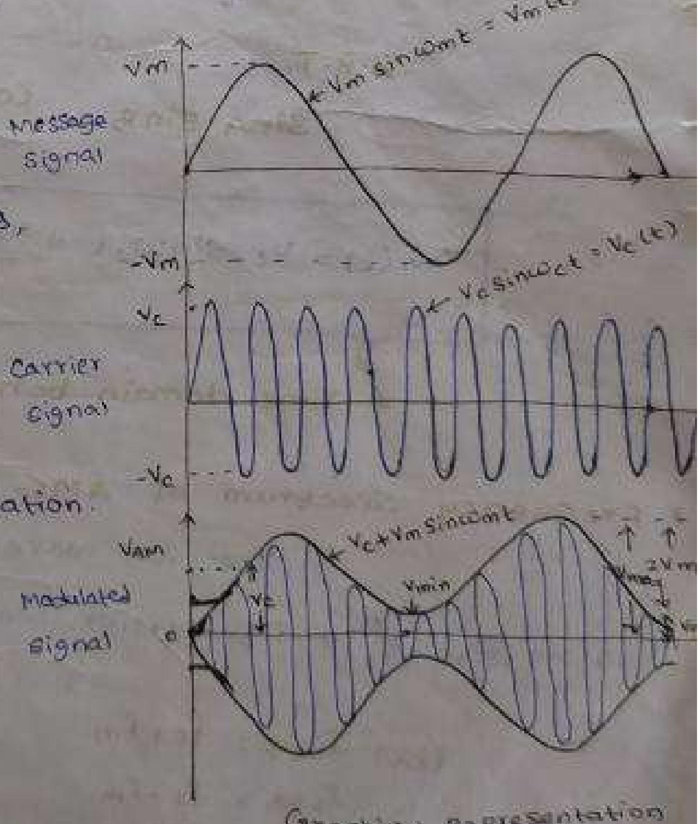
$V_{AM} = V_c + V_m(t)$ — ③

Sub. ① in ③

$$V_{AM} = V_c + V_m \sin\omega_m t$$

$$= V_c \left[1 + \frac{V_m}{V_c} \sin\omega_m t\right]$$

$$\boxed{V_{AM} = V_c \left[1 + m_a \sin\omega_m t\right]} \quad \text{— ④}$$



Graphical Representation of AM envelope.

Since, $m_a = \frac{V_m}{V_c}$ → modulation index (or) depth of modulation $= \frac{\text{Maximum amplitude of modulating signal}}{\text{max. amplitude of carrier signal}}$

where,

$V_m \rightarrow$ Max. amplitude of modulating signal

$V_c \rightarrow$ Max. amplitude of carrier signal

$\omega_m \rightarrow$ angular freq. of modulating signal

$\omega_c \rightarrow$ angular freq. of carrier signal.

Shape of modulated signal in AM envelope because, it contain all frequencies that make up the AM signal and it used to communicate the information through the system.

The instantaneous amplitude of modulated signal or AM envelope can be written as,

$$V_{AM}(t) = V_{AM} \sin\omega_c t \quad \text{—— (5)}$$

sub (4) in (5), we get,

$$V_{AM}(t) = V_c [1 + m_a \sin\omega_m t] \sin\omega_c t$$

$$= V_c \sin\omega_c t + m_a V_c \sin\omega_m t \sin\omega_c t$$

W.K.T,

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$\boxed{V_{AM}(t) = V_c \sin\omega_c t + \frac{m_a V_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]} \quad \text{—— (6)}$$

Time domain behaviour of AM signal.

* Frequency spectrum of AM:-

The AM wave contains three terms in EQ. no. (6), $1^{st} \rightarrow$ full carrier wave; $2^{nd} \rightarrow$ Lower side band (LSB), $3^{rd} \rightarrow$ Upper side band (USB)

(i,e) $f_{USB} = f_c + f_m$

$f_{LSB} = f_c - f_m$

where, $f_c \rightarrow$ carrier frequency

$f_m \rightarrow$ modulating frequency.

By considering AM wave,

$$V_{AM}(t) = V_c \sin\omega_c t + \frac{m_a V_c}{2}\cos(\omega_c - \omega_m)t - \frac{m_a V_c}{2}\cos(\omega_c + \omega_m)t$$

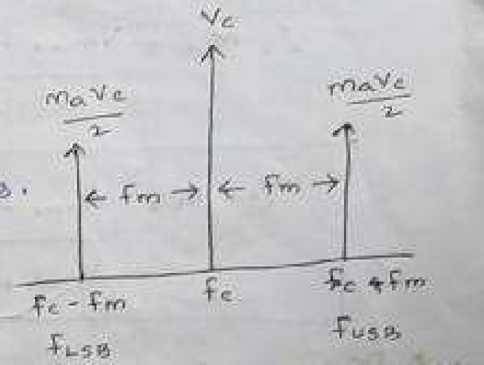
$$\left[\because \omega_c = 2\pi f_c, \quad \omega_m = 2\pi f_m\right]$$

$$V_{AM}(t) = V_c \sin 2\pi f_c t + \frac{m_a V_c}{2} \cos 2\pi (f_c - f_m)t - \frac{m_a V_c}{2} \cos 2\pi (f_c + f_m)$$

$$\therefore V_{AM}(t) = V_c \sin 2\pi f_c t + \frac{m_a V_c}{2} \cos 2\pi f_{LSB} t + \frac{m_a V_c}{2} \cos 2\pi f_{USB} t$$ //

* Bandwidth:-

Bandwidth of AM can be obtained by taking the difference between highest and lowest frequencies.



Frequency domain representation of AM wave.

$$\left.\begin{array}{c}\text{Bandwidth} \\ (B \cdot W)\end{array}\right\} = f_{USB} - f_{LSB}$$

$$= (f_c + f_m) - (f_c - f_m)$$

$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$

$$\boxed{B \cdot W = 2 f_m}$$

(ie), BW of AM is twice of the maximum frequency of modulating signal.

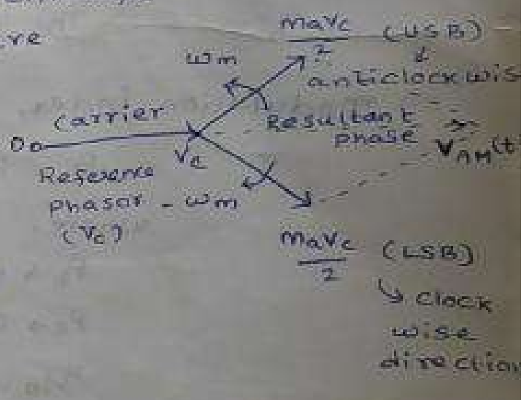
* Phasor (or) Vector representation of AM with carrier:-

→ maximum positive amplitude of envelope occurs if V_c, V_{LSB}, V_{USB} all are have positive values or inphase.

ie, $V_{max} = V_c + V_{LSB} + V_{USB}$

→ minimum positive amplitude of envelope occurs if V_c, V_{LSB}, V_{USB} all are in out of phase

ie, $V_{min} = V_c - V_{LSB} - V_{USB}$.



* Modulation index (or) co-efficient of modulation (or) Percent modulation:-

Definition:- Modulation index "m_a" is used to describe the "amount of amplitude change" occur in AM envelope"; $m_a = \frac{V_m}{V_c}$

from envelope, $2V_m = V_{max} - V_{min}$

$$\boxed{V_m = \frac{V_{max} - V_{min}}{2}}$$

&

$$(V_{carrier})_{max} = V_{max} - V_m$$

$$= V_{max} - \left(\frac{V_{max} - V_{min}}{2}\right)$$

$$\boxed{V_c = \frac{V_{max} + V_{min}}{2}}$$

$$m_a = \frac{V_m}{V_c}$$

Sub V_m & V_c in m_a;

$$\therefore m_a = \frac{V_{max} - V_{min}}{\cancel{2}} \times \frac{\cancel{2}}{V_{max} + V_{min}}$$

$$\boxed{\% \, m_a = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \times 100}$$

* Degree of modulation:-

The degree of modulation depending up on the amplitude of message signal relative to carrier amplitude.

i) under modulation $m_a < 1$; $V_m < V_c$

ii) Critical modulation $m_a = 1$; $V_m = V_c$

iii) Over modulation $m_a > 1$; $V_m > V_c$

* AM Power distribution:-

The total Power in the modulated wave depends on modulation index,

$$P_t = P_c + P_{LSB} + P_{USB}$$

where,

$P_t \rightarrow$ total power

$P_c \rightarrow$ Carrier Power

$P_{LSB,USB} \rightarrow$ Power in two side bands.

* Note:
W.K.T, By ohm's law
$V = IR$
Then, Power.
$P = IV$
$P = I^2 R$
$P = V^2/R$ ✓

$$\therefore P_t = \frac{V_c^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

where, $V_{carrier} \rightarrow$ RMS value of Carrier voltage.

$|||^{ly}$ $V_{LSB,USB} \rightarrow$ R.M.S. value of side band voltages

$R \rightarrow$ Resistance in which power is dissipated.

* Note:
$$R.M.S = \frac{Amplitude}{\sqrt{2}}$$

$$\therefore P_c = \frac{V_c^2}{R} = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \frac{\left(\frac{m_a V_c}{2}/\sqrt{2}\right)^2}{R} = \frac{\frac{m_a^2 V_c^2}{4}/2}{R}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2 V_c^2}{8R}$$

$\therefore$ Total power $P_t = \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R}$

$$= \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{4R}$$

$$P_t = \frac{V_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right] \qquad \left[\because P_c = \frac{V_c^2}{2R}\right]$$

$$\boxed{P_t = P_c\left[1 + \frac{m_a^2}{2}\right]}$$

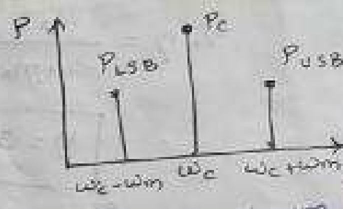
(or)

$$\frac{P_t}{P_c} = 1 + \frac{m_a^2}{2} \quad \text{(or)} \quad m_a = \sqrt{2\left(\frac{P_t}{P_c} - 1\right)}$$

(In terms of m_a)

if $m_a = 1$; ie $\cdot/.100$ modulation

$$\frac{P_t}{P_c} = 1.5 \; ; \; \boxed{P_t = 1.5\, P_c}$$



(power spectrum of AM)

* AM current Relations:-

W.K.T $P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$ —①

[$\because$ power in terms of current is $P_t = I_t^2 R$]
$P_c = I_c^2 \cdot R$

$$\therefore P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$$

$$I_t^2 \cdot \cancel{R} = I_c^2 \cancel{R}\left[1 + \frac{m_a^2}{2}\right]$$

$$\therefore \boxed{I_{total} = I_c\sqrt{1 + \frac{m_a^2}{2}}}$$

(ie) related to m_a;

$$\frac{I_{total}^2}{I_c^2} = 1 + \frac{m_a^2}{2}$$

$$m_a^2 = 2\left[\frac{I_{total}^2}{I_c^2} - 1\right]$$

$$\boxed{m_a = \sqrt{2\left(\frac{I_t^2}{I_c^2} - 1\right)}}$$

* Transmission Efficiency:-

$$\%\,\eta = \frac{\text{Power in side band}}{\text{Total Power}} \times 100\,\%$$

$$= \frac{P_{LSB} + P_{USB}}{P_t} \times 100$$

$$= \frac{\frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R}}{\frac{V_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right]}$$

$$= \frac{\frac{\cancel{2} m_a^2 V_c^2}{4 \cancel{8} R}}{P_c\left(1+\frac{m_a^2}{2}\right)} \times 100$$

$$= \frac{\frac{m_a^2}{2} \cdot \frac{V_c^2}{2R}}{P_c\left(1+\frac{m_a^2}{2}\right)} \times 100$$

$$= \frac{\frac{m_a^2 \cancel{P_c}}{2}}{\cancel{P_c}\left(1+\frac{m_a^2}{2}\right)} \times 100 = \frac{m_a^2}{\cancel{2}} \times \frac{\cancel{2}}{2+m_a^2} \times 100$$

$$\% \eta = \frac{m_a^2}{2+m_a^2} \times 100 \%$$ if $m_a = 1$; $\eta = 1/3 \times 100$

$$\boxed{\eta = 33.3 \%}$$

only 33.3 % of energy is used and remaining power is wasted by the carrier along with sidebands.

AMPLITUDE MODULATION BY MULTIPLE SINE WAVES :-

Consider, two modulating wave, let see what will happen if two or more sine wave modulate the carrier simultaneous

Two sine wave can be represented as,

$$\left.\begin{aligned} V_{m1}(t) &= V_{m1} \sin \omega_{m1} t \\ V_{m2}(t) &= V_{m2} \sin \omega_{m2} t \end{aligned}\right\} \quad \text{--- (1)}$$

The total modulating signal will be,

$$V_m(t) = V_{m1}(t) + V_{m2}(t)$$

$$V_m(t) = V_{m1} \sin \omega_{m1} t + V_{m2} \sin \omega_{m2} t \quad \text{--- (2)}$$

Let carrier signal is,

$$V_c(t) = V_c \sin \omega_c t \quad \text{--- (3)}$$

According to definition,

$$V_{AM} = V_c + V_m(t) \quad \text{--- (4)}$$

$$V_{AM} = V_c + V_{m1} \sin \omega_{m1} t + V_{m2} \sin \omega_{m2} t \quad \text{--- (5)}$$

∴ The instantaneous amplitude of modulated signal is,

$$V_{AM}(t) = V_{AM} \sin \omega_c t \quad \text{--- (6)}$$

sub (5) in (6)

$$V_{AM}(t) = (V_c + V_{m1} \sin \omega_{m1} t + V_{m2} \sin \omega_{m2} t) \cdot \sin \omega_c t \quad \text{--- (7)}$$

$$= V_c \left(1 + \frac{V_{m1}}{V_c} \sin \omega_{m1} t + \frac{V_{m2}}{V_c} \sin \omega_{m2} t\right) \sin \omega_c t$$

Let, $m_1 = \frac{V_{m1}}{V_c}$ & $m_2 = \frac{V_{m2}}{V_c}$

$$\therefore V_{am}(t) = V_c (1 + m_1 \sin \omega_{m_1} t + m_2 \sin \omega_{m2} t) \cdot \sin \omega_c t$$

$$V_{am}(t) = V_c \sin \omega_c t + m_1 V_c \sin \omega_{m1} t \sin \omega_c t + m_2 V_c \sin \omega_{m2} \sin \omega_c t$$

III^{ly}

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

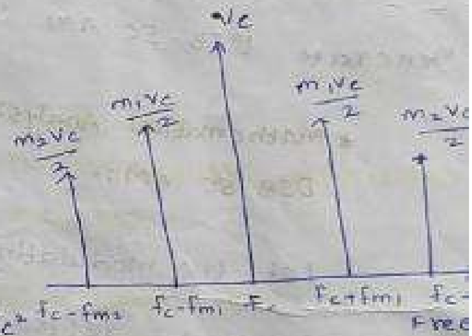
$$V_{AM}(t) = V_c \sin \omega_c t + \frac{m_1 V_c}{2} \cos(\omega_c - \omega_{m_1})t - \frac{m_1 V_c}{2} \cos(\omega_c + \omega_{m_1})t$$
$$+ \frac{m_2 V_c}{2} \cos(\omega_c - \omega_{m2})t - \frac{m_2 V_c}{2} \cos(\omega_c + \omega_{m2})t$$

Time domain behaviour of AM with multiple sine waves

* Power distribution :-

The total power,

$$P_t = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$



Frequency domain representat

W.K.T

$$P_t = \frac{V_c^2}{2R} + \frac{m_1^2 V_c^2}{8R} + \frac{m_2^2 V_c^2}{8R} + \frac{m_1^2 V_c^2}{8R} + \frac{m_2^2 V_c^2}{8R}$$

$$P_t = \frac{V_c^2}{2R}\left[\frac{m_1^2}{4} + \frac{m_2^2}{4} + \frac{m_1^2}{4} + \frac{m_2^2}{4} + 1\right]$$

$$P_t = \frac{V_c^2}{2R}\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2}\right] \quad \because P_c = \frac{V_c^2}{2R}$$

$$\boxed{P_t = P_c \left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2}\right]}$$

For multiple sinewave, $P_{total} = P_c \left(1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2} + \cdots\right)$

Consider, $\frac{m_t^2}{2} = \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2} + \cdots$

$$\boxed{P_{total} = P_c \left(1 + \frac{m_t^2}{2}\right)}$$

Total modulation index $m_t = \sqrt{m_1^2 + m_2^2 \cdots}$

* Current Relation :-

$$I_{total} = I_c \sqrt{1 + \frac{m_t^2}{2}}$$

* Transmission Efficiency.

$$\% \eta = \frac{\text{Power in side band}}{\text{Total Power}}$$

$$\% \eta = \frac{m_t^2}{m_t^2 + 2} \times 100 \%$$

(2) Double Side Band - Suppressed Carrier (DSB-SC) - AM :-

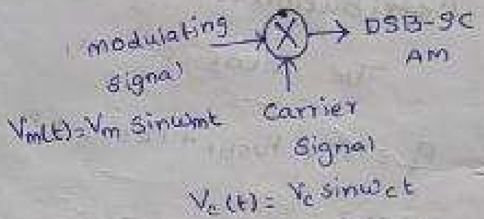
In Amplitude modulation (DSB-FC) wave has three Components,

i) Carrier ii) Upper side band (USB) iii) Lower side band (LSB)

Here Power wastage in transmission and bandwidth is wasted. The carrier signal in DSBFC system does not convey any information. So, the carrier is suppressed to save the power.

In DSB-SC, it contain only LSB and USB, resulting that transmission bandwidth is twice the frequency of modulating signal.

Basic concept :- Product multiplier (Balanced modulator) used to generate DSB-SC AM signal.



* Mathematical Analysis of DSB-SC AM :-

Let the modulating and carrier signal be represented as,

$$V_m(t) = V_m \sin\omega_m t$$

$$V_c(t) = V_c \sin\omega_c t$$

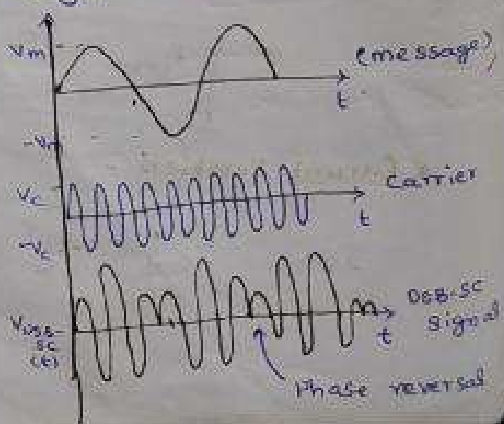
$$\therefore \; V(t)_{DSB-SC} = V_m(t) \cdot V_c(t)$$

$$\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$= V_m \sin\omega_m t \; V_c \sin\omega_c t$$

$$V(t)_{DSB-SC} = \frac{V_m V_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

By comparing the above equation with DSB-FC, only two side band present, $V_c \sin\omega_c t$ is neglected. Hence it is called DSB-SC.



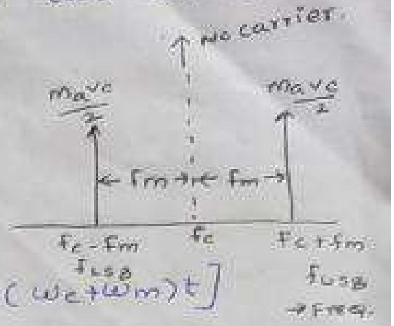
* Frequency Spectrum of DSB-SC AM:-

The DSB-SC AM contains two terms

(ie) Upper side band and Lower side band.

(ie) $f_{USB} = f_c + f_m$

$f_{LSB} = f_c - f_m$



By considering DSB-SC-AM wave,

$$V(t)_{DSB\text{-}SC} = \frac{V_m V_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

$\omega = 2\pi f$

* Bandwidth of DSB-SC AM:-

Band width can be obtained by taking the difference between highest and lower frequency in frequency spectrum.

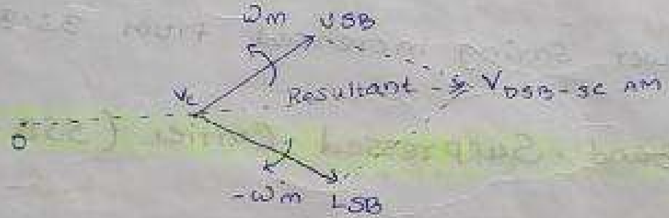
$$\text{Bandwidth} = f_{USB} - f_{LSB}$$

$$= f_c + f_m - (f_c - f_m)$$

$$\boxed{B.W = 2f_m}$$

$\therefore$ B.W is same as the DSB FC AM signal.

* Phasor Representation:-



* Power Distribution:-

From DSB-FC, W.K.T $P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$

if carrier is suppressed, then the power transmit in DSB-SC AM is termed as,

$$P_t' = P_{LSB} + P_{USB} \quad \text{—(1)}$$

$$P_t' = \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

where,

$V_{USB} = V_{LSB}$ = R.M.S value of side band voltages

$$\therefore P_{LSB} = P_{USB} = P_{SB} = \frac{V_{SB}^2}{R} = \frac{\left(\frac{m_a V_c/2}{\sqrt{2}}\right)^2}{R} = \frac{m_a^2 V_c^2}{8R} \rightarrow (2)$$

sub (2) in (1)

Total Power in DSB-SC

$$\therefore P_t' = \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R}$$

$$= \frac{\cancel{2} m_a^2 V_c^2}{4\cancel{8}R} \qquad \left[\because P_c = \frac{V_c^2}{2R}\right]$$

P_t

$$= \frac{\left[1+\frac{m_a^2}{2}\right]P_c - \frac{1}{2}m_a^2 \cdot P_c}{\left[1+\frac{m_a^2}{2}\right]P_c} \times 100$$

$$= \frac{\cancel{P_c}\left[1+\cancel{\frac{m_a^2}{2}} - \cancel{\frac{m_a^2}{2}}\right]}{\cancel{P_c}\left[1+\frac{m_a^2}{2}\right]} \times 100$$

$$\eta = \frac{2}{2+m_a^2} \times 100 \qquad \text{if } m_a = 1;$$

$$\eta = \frac{2}{3} \times 100$$

$$\boxed{\eta = 66.7\%}$$ $\therefore$ More power is saved than the DSB-FC-AM.

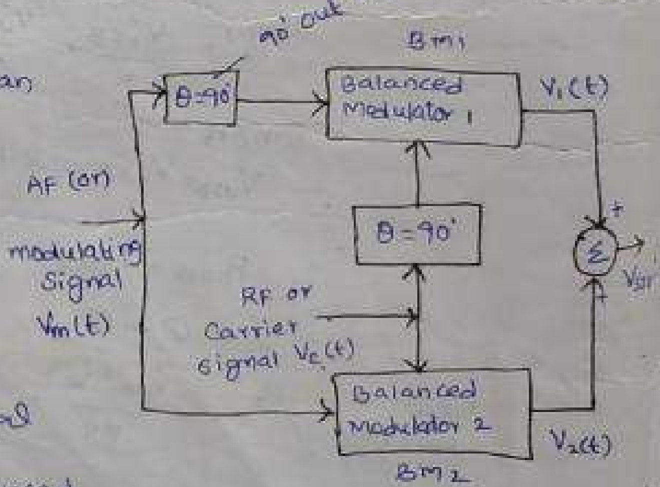
Here power saving increased from 33.3% to 66.7%.

(3) Single Side Band - Suppressed Carrier (SSB-SC-AM):- ✓

In DSB-SC power is saved but 66.7% power only but bandwidth are same (ie) $2f_m$.

By using the method SSB-SC, the power is saved more and bandwidth is reduced to half of it than the DSB-SC.

From the figure, we can obtain a SSB-SC by using 90° out of phase the message signal and carrier signal of balanced modulator 1. Then modulating and carrier signal is directly apply to the Balanced modulator 2. Because of 90° phase shift signal (ie) one



* Basic concept of SSB-SC

So

* F

of the side band is cancelled it. Therefore, the modulated signal of SSB (i.e.) $V_{SSB-SC}(t)$ contains only one side band it may be either lower side band or upper side band.

The mathematical analysis of SSB-SC is represented as, From Figure.

Let $v_m(t) = V_m \sin\omega_m t$

$v_c(t) = V_c \sin\omega_c t$

The output of B.M 1.

$$v_1(t) = V_m \sin(\omega_m t + 90^\circ)t \cdot V_c \sin(\omega_c t + 90^\circ)t$$

$[\because \sin(90+\theta) = \cos\theta]$

$$\therefore v_1(t) = V_m V_c \cos\omega_m t \cos\omega_c t \quad \text{--- (1)}$$

The output of BM 2,

$$v_2(t) = V_m \sin\omega_m t \; V_c \sin\omega_c t.$$

$\therefore$ Modulated signal of SSB-SC-AM is,

$$V_{SSB-SC}(t) = v_1(t) + v_2(t)$$

$$= V_m V_c \cos\omega_m t \cos\omega_c t + V_m V_c \sin\omega_m t \sin\omega_c t$$

$$= V_m V_c \left[\sin\omega_m t \sin\omega_c t + \cos\omega_m t \cos\omega_c t\right]$$

$$\left[\because \sin A \sin B + \cos A \cos B = \frac{\cos(A-B)}{2}\right]$$

$$\boxed{V_{SSB-SC} = \frac{V_m V_c}{2} \cos(\omega_c - \omega_m)t}$$

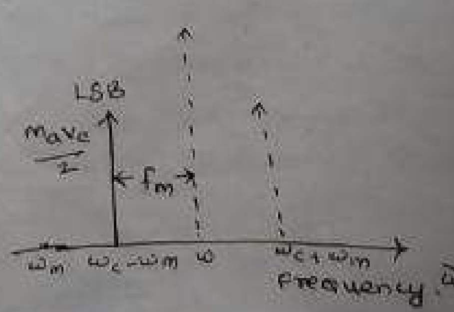
$\therefore$ The above equation present only one side band, single sideband suppressed carrier (SSB-SC-AM). So it is known as single sideband suppressed carrier (SSB-SC-AM).

* Frequency spectrum of BSB-SC-AM:-

$f_{LSB} = f_c - f_m$

By considering SSB-SC-AM

$$v(t)_{SSB-SC} = \frac{V_m V_c}{2} \cos(\omega_c - \omega_m)t.$$

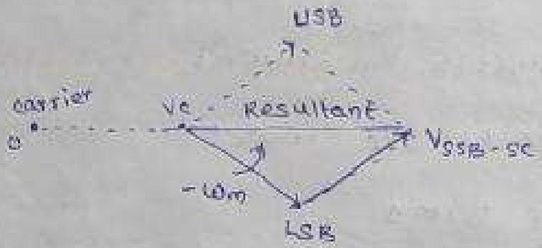


* Bandwidth of SSB-SC-AM:-

The SSB-SC-AM contains one side band, the bandwidth can be obtained by taking only one side band, it contains only f_m.

$$\boxed{B.W = f_m}$$

* Phasor Representation:-



* Power Distribution:-

From AM, $P_t = P_c\left(1+\frac{m_a^2}{2}\right)$

W.K.T

if both the carrier and one of the sideband is suppressed, only one side band is transmitted. Therefore the total power is represented as,

$$P_t'' = P_{LSB} = \frac{V_{LSB}^2}{R}$$

where,

$V_{LSB} \rightarrow$ R.M.S value of side band voltage (lower)

$$\therefore P_{LSB} = \frac{\left(\frac{\frac{m_a V_c}{2}}{\sqrt{2}}\right)^2}{R}$$

$$P_{LSB} = \frac{m_a^2 V_c^2}{8R}$$

$$\therefore P_t'' = \frac{m_a^2}{4} \cdot \frac{V_c^2}{2R} \qquad \left[\because P_c = \frac{V_c^2}{2R}\right]$$

$$\boxed{P_t'' = \frac{m_a^2}{4} \cdot P_c}$$

* Transmitted Efficiency:-

$$\eta = \frac{P_t - P_t''}{P_t} = \frac{P_c\left(1+\frac{m_a^2}{2}\right) - \frac{m_a^2}{4}\cdot P_c}{P_c\left(1+\frac{m_a^2}{2}\right)}$$

$$= \frac{\cancel{P_c}\left[1+\frac{m_a^2}{2} - \frac{m_a^2}{4}\right]}{\cancel{P_c}\left[1+\frac{m_a^2}{2}\right]}$$

$$\eta = \frac{\left(1+\frac{m_a^2}{4}\right)}{\left(1+\frac{m_a^2}{2}\right)} = \frac{4+m_a^2}{4\,2} \times \frac{2}{2+m_a^2}$$

$$\eta = \frac{4+m_a^2}{4+2m_a^2}$$ if $m_a = 1$; then,

$$\eta = \frac{5}{6} \times 100\% \quad \boxed{\eta = 83.3\%}$$

Saving of Power in SSBSC with respect to DSB-SC is,

$$\eta = \frac{P_t' - P_t''}{P_t'}$$; we get $\boxed{\eta = 50\%}$

1. GENERATION AND DETECTION OF AM (AMPLITUDE MODULATION)

(Modulator)

GENERATION OF AM:-

The device which is used to generate AM wave is known as amplitude modulator.

AM Generation

Low level modulation	High level Modulation
(or)	(or)
Non-linear Modulation	Linear Modulation
(or)	(or)
Small signal modulators	Large signal modulators

Input → Kept small to device

only used for small signal m

Low level modulation:

eg. FET, transistor, Diode.

Definition:- These modulator make use of non-linear (V-I) characteristics of the device. Devices are operated in non-linear region.

TYPES:

(i) Square law modulator

(ii) Product Modulator

(iii) Balanced modulator.

Drawback:-

→ Heavy filtering is required to remove unwanted signal present in o/p.

→ Amplification is used to power up desired level, because it is low voltage.

... this Problem, linear method used.

High level modulation:

Definition: The device are operated in linear region of its transfer characteristics thus the relation between the amplitude of modulating signal and depth of modulation is linear.

(Here also use pair of non-linear element having same characteristics in a balanced ckt. so that unwanted sgl. cancel out.

TYPES

(i) Transistor → Emitter, Base, Collector

(ii) Switching modulator

* Comparison Between Linear and Non-Linear Modula[tors]

Linear modulators	Non-Linear Modulators
1) Heavy filtering is not required while extracting the desired modulated frequency.	1) Heavy filtering is required for extractin[g] the desired modulate[d] frequency terms.
2) These modulators are used in High Level Modulation.	2) These modulators a[re] used in low level modulation.
3) The carrier voltage is very much greater than the modulating signal voltage (ie) $V_c >> V_m$	3) The modulating si[gnal] voltage is very much greater than carrier vo[ltage] (ie) $V_m >> V_c$

NOTE:- Generally, semiconductor devices such as diode, BJT or JFET may be used for generating the modulating signal.

NON-LINEAR MODULATION.

(i) SQUARE LAW MODULATOR :-

or (POWER LAW MODULATOR)

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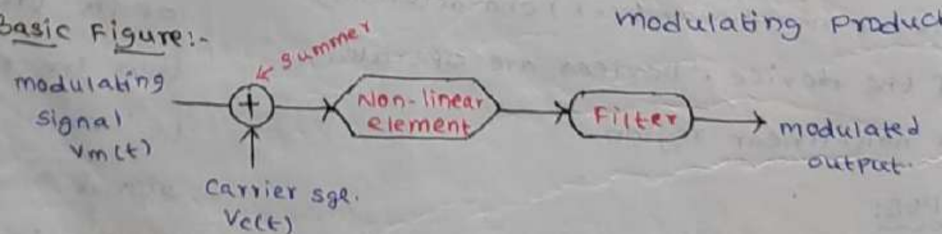
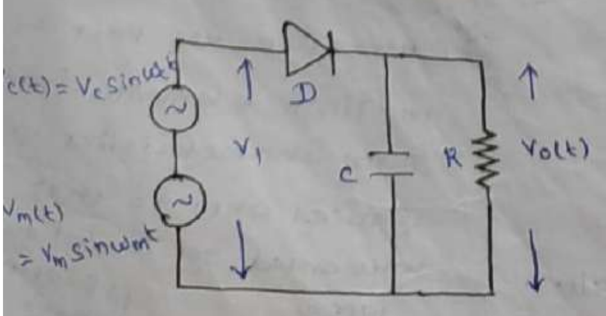
It has three features,

(1) Summer → To sum carrier & modulating signal
(2) Non-linear (active) element. (diode, BJT, JFET)
(3) BPF (Band Pass Filter) → For extracting desir[ed] modulating product.

Basic Figure:-





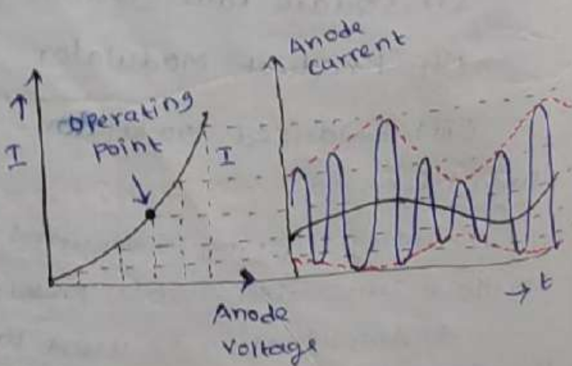


Fig a) Square law modulator

*Operation:-

The message signal (A.F) and carrier signal (R.F) applied at the input, are super imposed each other and makes the diode more forward bias during +ve half cycle of input and less forward biased during negative half cycle.

Thus the magnitude of the carrier component is greater during +ve cycle of modulating voltage and less during -ve sgl is shown in fig. b).

*Analysis:-

The resulting ~~current is~~ voltage will be given by square law equation is,

(i,e) $V_0 = a_1 V_1(t) + a_2 V_1^2(t) + \cdots$ —①

$V_1 \rightarrow$ Vgl applied to diode or FET

$V_1 = V_m \sin \omega_m t + V_c \sin \omega_c t$ —②

② in ①

$V_0(t) = a_1[V_m \sin \omega_m t + V_c \sin \omega_c t] + a_2[V_m \sin \omega_m t + V_c \sin \omega_c t]^2$

$\because$ expand $[a+b]^2 = a^2+b^2+2ab$

$V_0(t) = a_1 V_m \sin \omega_m t + a_1 V_c \sin \omega_c t + a_2 V_m^2 \sin^2 \omega_m t + a_2 V_c^2 \sin^2 \omega_c t + 2 a_2 V_m V_c \sin \omega_c t \sin \omega_m t$

Neglecting DC, 2nd & higher order terms, we get,
(BPF tuned to the freq of f_c, $f_c - f_m$ & $f_c + f_m$)

$V_0(t) = a_1 V_m \sin \omega_m t + a_1 V_c \sin \omega_c t + 2 a_2 V_m V_c \sin \omega_c t \sin \omega_m t$

$\because$ Expand $\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$

$i_0(t) = a_1 V_m \sin \omega_m t + a_1 V_c \sin \omega_c t + \frac{\cancel{2} a_2 V_m V_c}{\cancel{2}}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$

When the BPF is tuned carrier frequency, it allows only ω_c, $(\omega_c - \omega_m)$ & $(\omega_c + \omega_m)$ terms and it eliminates all other terms,

$i_0(t) = a_1 V_c \sin \omega_c t + a_2 V_m V_c \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$

Hence AM wave is generated.

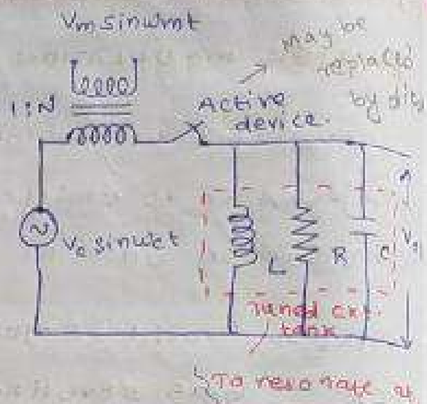
Draw back:

Single diode is unable to balance out desired frequency. It does not provide amplification. These can be eliminated by using amplifying device transistor, FET replacing dio

(ii) SWITCHING MODULATOR (or) CHOPPER MODULATOR:-

P17

Operation:-



Let $V_i(t) = V_c(t) + V_m(t)$

I/P to switching device.

Since the message signal is weak in magnitude, the polarity of weak signal depends on the polarity of carrier signal.

* The switching modulator is used

message signal → very weak & carrier → large magnitude. f_c -

The diode is forward bias for every +ve cycle of carrier & behaves likes a short circuit switch.

$$\therefore V_o(t) = V_i(t).$$

* For -ve cycle of carrier, the diode is reverse bias & open switch. The signal does not reach the filter & no output is obtained.

$$\text{(ie)}\ V_o(t) = 0.$$

* The o/p of BPF is tuned to carrier frequency hence it allows all V_c terms & reject other frequency terms.

$$\left.\begin{aligned} V_o(t) &= V_i(t)\ (1) \quad \text{for } V_c(t) > 0 \\ &= V_i(t)\ (0) \quad \text{for } V_c(t) < 0 \end{aligned}\right\} \text{— (1)}$$

In general above eq. written as,

$$V_o(t) = V_i(t) \cdot g(t) \qquad ; \ g(t) = \text{train of pulse}$$

Fourier series representation of $g(t)$ can written as,

$$\begin{cases} 1 \ ; \ V_c(t) > 0 \\ 0 \ ; \ V_c(t) < 0 \end{cases}$$

$$g(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)} \cos\left[2\omega_c(n-1)t\right]$$

$$\therefore V_o(t) = \frac{V_c}{2}\left[1 + m_a N \sin\omega_m t\right] \sin\omega_c t.$$

$$V_o(t) = \frac{V_c}{2}\sin\omega_c t + \frac{V_c}{2} m_a N \sin\omega_m t \sin\omega_c t \text{ — (2)}$$

Comparing AM with above eq.

$$\left[\because m_a = V_m\right]$$

$$= \frac{V_c}{2}\sin\omega_c t + \frac{V_m V_c \cdot N}{2}\left\{\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right\}$$

∴ AM is generated.

GENERATION OF DSB-SC-AM:- R201"

They are two ways to generate DSB-SC AM

a) Balanced Modulator (Heavy Filter is not required (non-linear device operate in balanced m

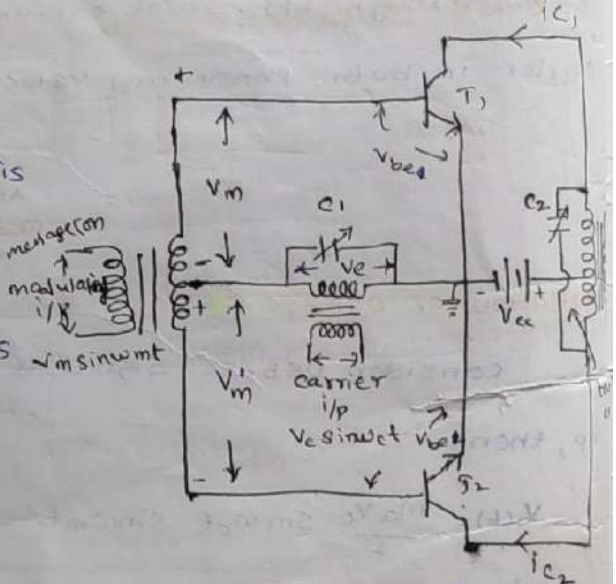
b) Ring Modulator

a) BALANCED MODULATOR:-

The same circuit can be used to generate AM with carrier. The main difference between AM with carrier generation and DSB-SC-AM is, the feeding point of carrier and modulating signal is interchanged.

* Operation:-

Two transistor are identical and the circuit is symmetrical. I/p is applied to differential i/p & carrier is applied to common i/p of pushpull amplifier configuration.



The signal at base of T_1 is sum of two input voltage (V_c+V_m) & T_2 is difference of two voltage (V_c-V_m). The input carrier is cancelled, as two collector current are subtracted in o/p transformer primary.

(To remove the carrier, 2 transistors are connected in balanced mode).

$$\left.\begin{aligned} V_{be_1} &= V_c + V_m = V_c \sin\omega_c t + V_m \sin\omega_m t \\ V_{be}' &= V_c - V_m = V_c \sin\omega_c t - V_m \sin\omega_m t \end{aligned}\right\} -①$$

The collector element is given by.

$$\left.\begin{aligned} i_{c_1} &= a_1 V_{be} + a_2 V_{be}^2 \\ i_{c_2} &= a_1 V_{be}' + a_2 V_{be}'^2 \end{aligned}\right\} -②$$

sub. ① in ②

$$i_{c_1} = a_1 V_c \sin\omega_c t + a_1 V_m \sin\omega_m t + a_2 V_c^2 \sin^2\omega_c t + a_2 V_m^2 \sin^2\omega_m t + 2a_2 V_m V_c \sin\omega_c t \sin\omega_m t$$

$$i_{c_2} = a_1 V_c \sin\omega_c t - a_1 V_m \sin\omega_m t + a_2 V_c^2 \sin^2\omega_c t + a_2 V_m^2 \sin^2\omega_m t - 2a_2 V_m V_c \sin\omega_c t \sin\omega_m t$$

$$\therefore V_0 = K(i_{c_1} - i_{c_2})$$

$$V_0 = 2K a_1 V_m \sin\omega_m t + 4 V_m V_c K \sin\omega_m t \sin\omega_c t$$

Tuning the tank ckt to carrier frequency & it respond to a band of frequency centered at ω_c

* Advantage:-

i) Economy of power; because of carrier is suppressed

ii) Increases the efficiency

iii) used in Telephony system, in which one side band is filtered out to reduce the width of the channel required for transmission.

iv) It offers secrecy.

b) RING MODULATOR (or) DIODE BALANCED MODULATOR:-

(Here we cannot use a BPF to remove unwanted modulating and carrier signal in its output). No filter.

A ring modulator uses 4 diodes is shown in fig.

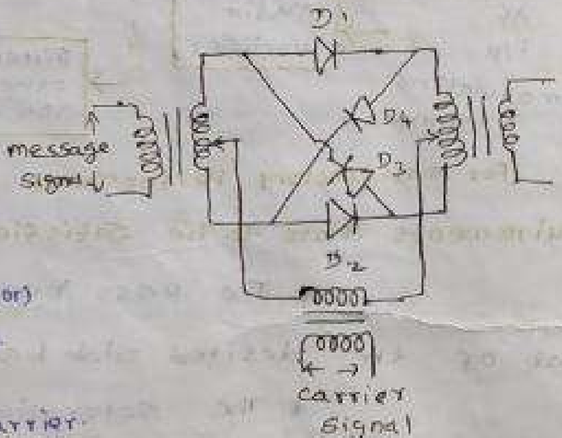


Fig: Ring modulator.

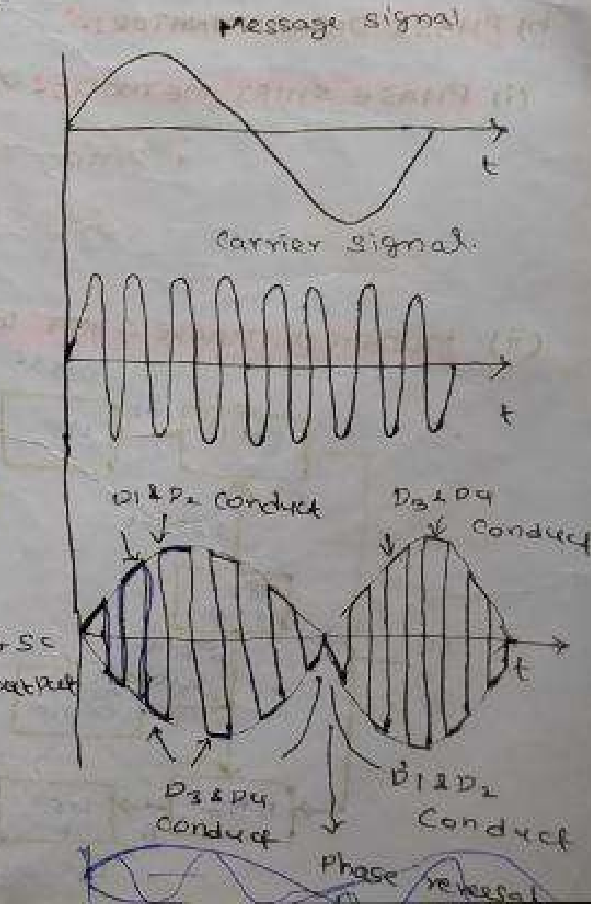
* Operation:-

No modulating signal → only carrier present → D_1 & D_2 (or) D_3 & D_4

Conduct depends on polarity of carrier.

output is zero.

when both signal are apply → +ve D_1 & D_2 conduct, -ve D_3 & D_4 conduct.



Consider,

$v_m(t) = V_m \sin\omega_m t$;

$v_c(t) = V_c \sin\omega_c t$

$\therefore v_o(t) = v_m(t) \cdot v_c(t)$

$$= \frac{V_m V_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

* Advantages:-

i) Output is stable

ii) Requires no external power source to activate the diode

iii) Longer life.

GENERATION OF SSB-SC AM

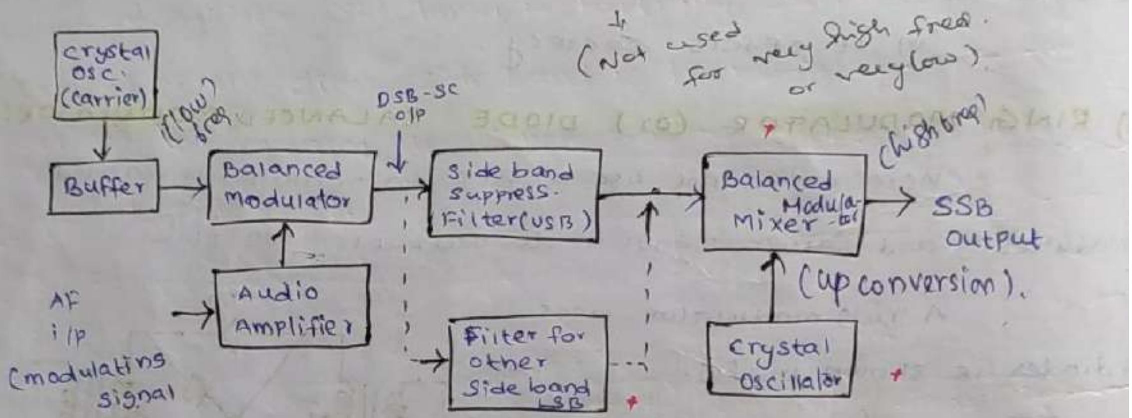
SSB-SC-AM waves can be generated in to two ways.

SSB-SC-AM

a) Frequency discriminator (or) Filter method (or) Selective Filtering Method

b) Phase Discriminator
   i) Phase shift method
   ii) Modified Phase shift (or) Weaver's Method

a) FILTER METHOD (or) FREQUENCY DISCRIMINATOR METHOD:-



For satisfactory performance of above system the following two requirements have to be satisfied,

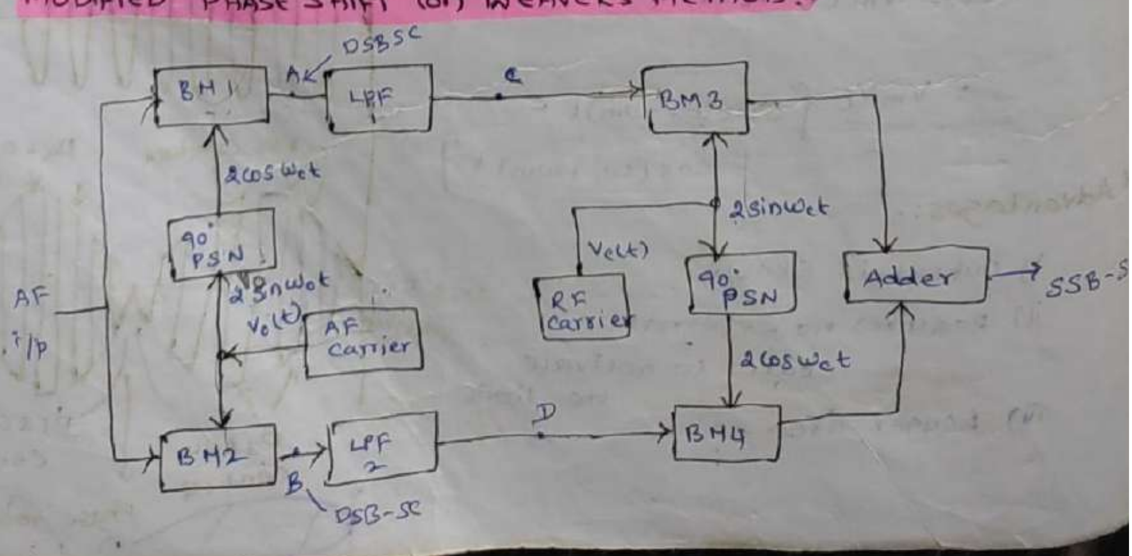
* The pass band of the filter should be same as that of the desired side band.
* The seperation region b/w pass band & stop band should not exceed twice the maximum frequency component present in base band. (ie) f_{PB} or $f_{SB} \leq 2f_m$

b) PHASE DISCRIMINATOR:-

(i) PHASE SHIFT METHOD:-

* P.no 1-7 is the answer for phase shift method.
(ie) SSB-SC derivation with block diagram.

(ii) MODIFIED PHASE SHIFT (or) WEAVER'S METHOD:-



Let modulating signal $V_m(t) = V_m \sin\omega_m t$

(Audio freq) AF sub carrier signal $V_o(t) = 2V_o \sin\omega_o t$

(Radio freq) RF sub carrier signal $V_c(t) = 2V_c \sin\omega_c t$

Step1: Input to BM1 = $V_m \sin\omega_m t \cdot 2V_o \sin(\omega_o t + 90°)$

output of BM1 = $V_m V_o [(\cos(\omega_o t - \omega_m t + 90°) - \cos(\omega_o t + \omega_m t + 90°)]$ — ①

Step2: $|||^{ly}$ BM2 output = $V_m V_o [\cos(\omega_o t - \omega m t) - \cos(\omega_o t + \omega m t)]$ — ②

Step3: LPF1 & LPF2 eliminate upper side band (USB) of BM1 & BM2 hence O/P of LPF1 & LP2 is,

$LPF\ 1 = V_m V_o \cos(\omega_o t - \omega_m t + 90°)$ — ③

$LPF\ 2 = V_m V_o \cos(\omega_o t - \omega_m t)$ — ④

Step4:- Assume $V_m = V_o = 1$

output of BM3 = $2\sin\omega_c t \cos(\omega_o t - \omega_m t + 90°)$

$[\because 2\sin A \cos B = \sin(A+B) \pm \sin(A-B)]$

BM3 = $\sin(\omega_c t + 90° + \omega_o t - \omega_m t) + \sin(\omega_c t - \omega_o t + \omega_m t - 90°)$

$|||^{ly}$,

BM4 = $2\sin(\omega_c t + 90°)\cos(\omega_o - \omega_m)t$

$= \sin(\omega_c t + \omega_o t - \omega_m t + 90°) + \sin(\omega_c t + 90° - \omega_o t + \omega m t)$

$\therefore$ Step5:

output of summer,

$= \sin((\omega_c + \omega_o - \omega_m)t + 90°) + \sin((\omega_c - \omega_o + \omega_m)t - 90°)$

$+ \sin((\omega_c + \omega_o - \omega_m)t + 90°) + \sin((\omega_c - \omega_o + \omega_m)t + 90°)$

$= 2\sin[(\omega_c + \omega_o - \omega_m)t + 90°]$

$= 2\cos(\omega_c + \omega_o - \omega_m)t$ //

O/P

Finally, RF output frequency is $f_c + f_o - f_m$ (LSB) of RF carrier $f_c + f_o$.

* Application of SSB-SC-AM:-

i) Police wireless communication

ii) Point to point radio telephone communication & mobile communication

iii) VHF & UHF communication system.

GENERATION AND DETECTION OF VSB:-

R17

VSB → VESTIGIAL SIDE BAND,

*DEFINITION:-

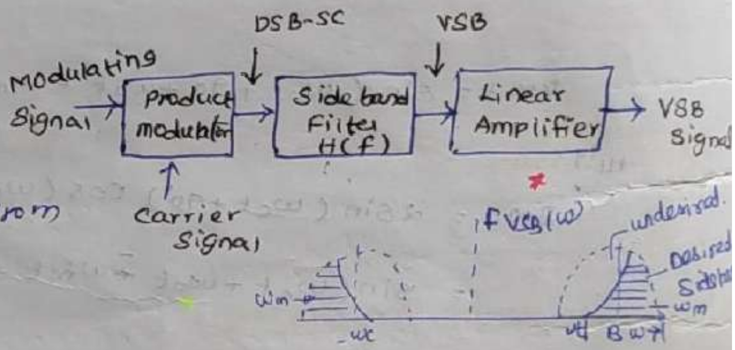
VSB is similar to SSB transmission, in which one of the side band is completely removed. In VSB, however the second side band is not completely removed, but it filtered to remove all but the desired range of frequency.

(or)

One of the side band is partially suppressed and vestige (portion) of the other side band is transmitted. This vestige (portion) compensates the suppression of sideband. It is called vestigial side band.

* VSB GENERATION:-

Let find characteristics of filter with a transfer function $H(f)$ that may generate VSB-AM signal from DSB-SC.



W.K.T

$$V_{DSB\text{-}SC}(t) = \frac{V_m V_c}{2}\left[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t\right]$$

The spectrum is given by,

$$\phi(\omega)_{DSB} = \frac{1}{2}F(\omega+\omega_c) + \frac{1}{2}F(\omega-\omega_c)$$

The output of filter,

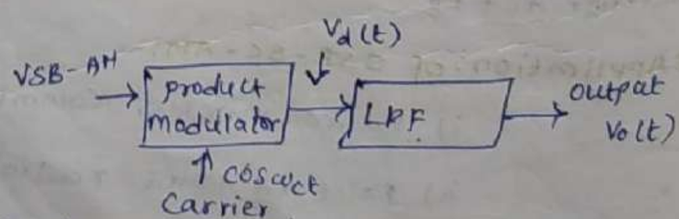
$$\phi_v(\omega) = \frac{1}{2}H(\omega)\left[F(\omega+\omega_c)+F(\omega-\omega_c)\right]$$

$\because H(\omega) \rightarrow$ Transfer function

Fourier Transform of $\phi_v(t) = \phi_v(\omega)$

* VSB DETECTION:-

$V_d(t) = \phi_v(t)\cdot\cos\omega_c t$



$$F[V_d(t)] = \frac{1}{2}\left[\phi_v(\omega+\omega_c) + \phi_v(\omega-\omega_c)\right] \quad (1)$$

sub $\phi_v(\omega)$ value in eq.(1)

$$V_d(t) = \frac{1}{2}\Big[\tfrac{1}{2}H(\omega+\omega_c)\left[F(\omega+\omega_c+\omega_c)+F(\omega+\omega_c-\omega_c)\right]$$
$$+\tfrac{1}{2}H(\omega-\omega_c)\left[F(\omega-\omega_c+\omega_c)+F(\omega-\omega_c-\omega_c)\right]\Big]$$

$$V_d(t) = \frac{1}{4}\Big[\{F(\omega+2\omega_c)+F(\omega)\}H(\omega+\omega_c) + \{F(\omega)+F(\omega-2\omega_c)\}H(\omega-\omega_c)\Big]$$

Then LPF, centered around $\pm 2\omega_c$ are filtered out,

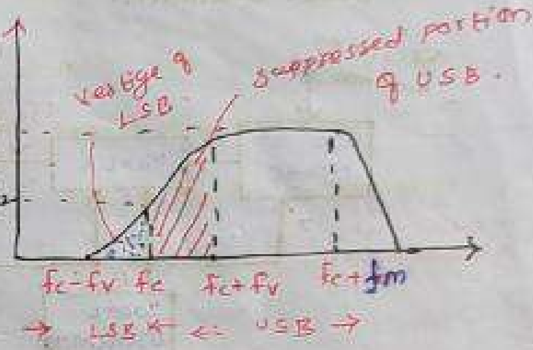
The output is

$$V_0(t) = C_1 F(\omega) \quad ; C_1 \to \text{constant}.$$

$$\therefore C_1 = H(\omega+\omega_c) + H(\omega-\omega_c).$$

$H(\omega)$ is shifted by $+\omega_c$ & $-\omega_c$; constant over range $|\omega| < \omega_m$.

* BANDWIDTH:-



$$B.W = f_{USB} - f_{LSB}$$
$$= f_c + f_m - (f_c - f_v)$$
$$= \cancel{f_c} + f_m - \cancel{f_c} + f_v$$
$$\boxed{B.W = f_v + f_m}$$

* APPLICATION:- used in TV transmission; (picture details)

Comparison:-

*COMPARISON OF AM SYSTEM:-

S.NO.	PARAMETER	AM with carrier DSB-FC	DSB-SC AM	SSB-SC AM	VSB-AM
1.	Method	Carrier & Both side band	only sideband	only one sideband	Vestige portion of side band
2.	Bandwidth	2fm	2fm	fm	fv + w
3.	Generation	Easy	Easy	Complex	Complex
4.	Power Saving	33.3%	66.7%	83.3%	83.3%
5.	Selective fading	Heavy distortion	more distortion compare to SSB	least distortion	Received signal is distorted
6.	Application	AM broadcast	Carrier telephony	police wireless, mobile	TV & high speed data Transmission

AM RECEIVER:-

Basically there are two types of receivers.

(1) TUNED RADIO FREQUENCY (TRF) RECEIVER

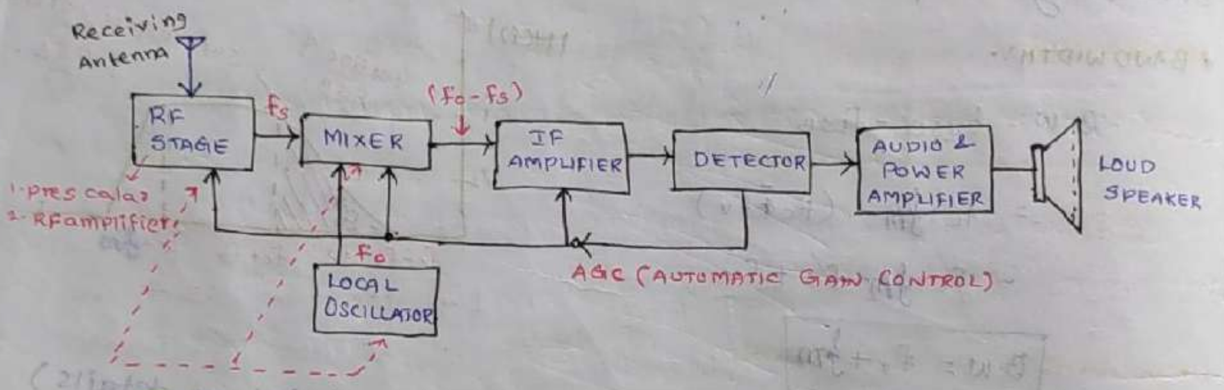
(2) SUPER HETERODYNE RECEIVER.

1. Gain is not uni (only for 2. low frequency application) 3. B.W varies

SUPERHETERODYNE RECEIVER:-

* **Heterodyne:-** Heterodyne means to mix two frequencies together in a non-linear manner or to translate one frequency to another using non-linear mixing.

* BLOCK DIAGRAM:-



* **PRINCIPLE:-**

The principle involves mixing the input signal with the local oscillator signal, and this is converted in to a signal of lower fixed frequency called "Intermediate frequency" which contains the same modulation level as the original received signal.

RF STAGE:-

✓ The incoming AM signal picked up by the received antenna is passed on to the RF section. This section consists of Prescalar and an RF amplifier.

1. Prescalar:- ✓ Prescalar is a BPF with an adjustable centre frequency that is tuned to desired carrier frequency of incoming si

✓ Main use of prescalar is to provide enough initial band limiting to prevent unwanted radio frequency signal called "Image frequency" and it also reduce B.W to improve SNR (signal to Noise Ratio).

2. RF amplifier:- It amplifies the incoming signal to desired signal.

↳ Advantage:- a) Greater Gain b) Better Sensitivity c) Better SNR d) Better Selectivity e) Better IFR (image frequency rejection)

FREQUENCY CHANGER:-

• The combination of mixer and local oscillator constitute the frequency changer. Both of them provide heterodyne function. The intermediate frequency is lower than the incoming carrier frequency. (ie) $f_{IF} = f_{RF} - f_{LO}$ (or) $f_{IF} = f_c - f_s$

/ The modulated signal is translated from RF to IF stage, the shape of the envelope and original information contained in the envelope remains unchanged.

IF SECTION:-

This section consists of one or more stages of tuned amplifier which amplifies the IF produced at the output of frequency changer circuit. This section provides most of the amplification and selectivity of the receiver.

DEMODULATOR (or) DETECTOR:-

The output of IF section is applied to detector which recovers the base band or message signal. If coherent detection is used, then local carrier source is provided in the receiver.

The detector also supplies d.c bias voltage to RF & IF stages in the form of AGC circuit. Finally the recovered signal is power amplified and enrouted to loud speaker.

ADVANTAGE OF SUPERHETERODYNE RECEIVER:-

i) Improved selectivity ii) Improved stability

iii) Higher gain per stage. iv) Uniform band width because of fixed IF frequency

PERFORMANCE PARAMETER OF RECEIVER:-

(1) Selectivity → measure of performance of radio Rx to respond only to a radio sgl is tuned & reject all sgl.

(2) Sensitivity → Rx or detection sgl provide minimum of i/p sgl required to produce specified o/p.

(3) Fidelity → Ability to accurately reproduce its output.

(4) IFR (Image Frequency Rejection) - reject interfering signal at the image frequency.

$$IFRR = \sqrt{1 + Q^2 \rho^2}$$

Q → Quality factor of prescalar

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

HILBERT TRANSFORM - PREENVELOPE AND COMPLEX ENVELOPE:-

Hilbert Transform:- It is a linear operator which takes a function and produce other function in the same domain.

For example, $\sin(t) \to -\cos(t)$

$\cos(t) \to \sin(t)$

$\exp(it) \to -i\exp(it)$

Hilbert transform of signal $x(t)$ is,

H.T $\to \hat{x}(t) = \frac{1}{\pi}\int_{-\alpha}^{\alpha} \frac{x(\tau)}{t-\tau}\cdot d\tau$ (Here both $x(t)$ & $\hat{x}(t)$ are in time domain)

I.H.T $\to x(t) = \frac{-1}{\pi}\int_{-\alpha}^{\alpha} \frac{\hat{x}(\tau)}{t-\tau}\cdot d\tau$

F.T. of Hilbert transform is, $\boxed{\hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f)}$

Property:-

1. Linearity $\to$ If A_1 & A_2 are arbitary scalar & G_1 & G_2 are signals $[a_1 g_1(t) + a_2 g_2(t)]^\wedge = a_1\hat{g}_1(t) + a_2\hat{g}_2$
2. Time shifting $\to g(t-t_0) = g(t-t_0)^\wedge$
3. Time dilation $\to g(at) \to \mathrm{sgn}(a)\, g^\wedge(at)$
4. Convolution $\to [g_1(t) * g_2(t)]^\wedge = g_1(t)^\wedge g_2(t)$
5. Time derivative. $H\left[\frac{d}{dt} g(t)\right] = \frac{d}{dt} H[g(t)]$

Pre-envelope & Complex Envelope:-

The pre envelope of sgl. $x(t)$ is,

$x_p(t) = x(t) + j\hat{x}(t)$

$\uparrow$ real part, $\uparrow$ HT of imaginary

F.T. of preenvelope

$X_p(f) = X(f) + j[-j\,\mathrm{sgn}(f)\,X(f)]$

$\therefore X_p(f) = X(f) + \mathrm{sgn}(f)\,X(f)$.

W.K.T $\mathrm{sgn}(f) = \begin{cases} 1 ; & f>0 \\ 0 ; & f=0 \\ -1 ; & f<0 \end{cases}$ hence,

$X_p(f) = \begin{cases} 2X(f) ; & f>0 \\ X(0) ; & f=0 \\ 0 ; & f<0 \end{cases}$

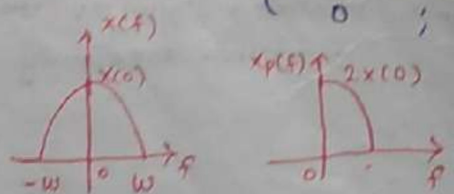


Fig. Pre envelope.

∴ No frequency content for -ve frequencies.

Complex Envelope:-

$x(t)$ is given as,

$x_c(t) = x_p(t)\, e^{-j2\pi f_c t}$

$x_p(t) = x_c(t)\cdot e^{j2\pi f_c t}$

F.T. of above eq. is

$\boxed{X_p(f) = X_c(f-f_c)}$

↳ low pass spectrum

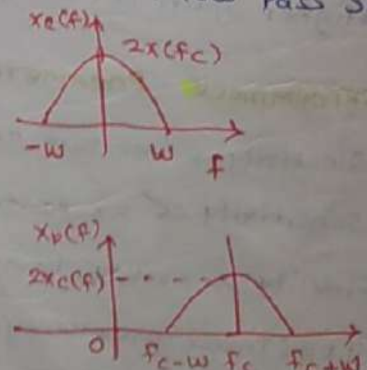


Fig. Complex envelope.

UNIT-2 ANGLE MODULATION

Phase and Frequency Modulation - Narrow Band and Wide band FM - Spectrum - FM modulation and Demodulation - FM Discriminator - PLL as FM Demodulation - Transmission Bandwidth.

ANGLE MODULATION :- (or) EXPONENTIAL MODULATION:-

Angle modulation is the process by which the angle (ie, Frequency or phase of the carrier signal is changed in accordance with the instantaneous amplitude of modulating or message signal, but the amplitude of the carrier remains constant.

Angle modulation is further divided in to two types.

* FREQUENCY MODULATION
* PHASE MODULATION

In general angle modulated wave is mathematically expressed as,

$$V(t) = V_c \sin(\omega_c t + \theta(t))$$

where, $\phi(t) = \omega_c t + \theta(t)$ → Phase angle of carrier signal.

$$\therefore \boxed{V(t) = V_c \sin \phi(t)}$$

Here,

$V_c \rightarrow$ Peak amplitude of carrier signal.

$\omega_c \rightarrow$ Angular frequency of carrier signal.

$\theta(t) \rightarrow$ Instantaneous phase deviation.

PHASE AND FREQUENCY MODULATION:-

* IMPORTANT DEFINITION TERMS:-

(1) Instantaneous phase "ϕ" :-

Instantaneous phase of the carrier is precise phase of the carrier at any instant of time. It is denoted by $\phi(t)$.

$$\boxed{\phi(t) = \omega_c t + \theta(t)} \text{ radians.}$$

(2) Instantaneous phase deviation $\theta(t)$:-

It change in phase of the carrier at any instant of time with respect to reference phase.

$$\theta(t) \propto V_m(t) \quad \text{(ie)} \quad \theta(t) = K \cdot V_m(t) \text{ radians}$$

Instantaneous Frequency:- (ω_i)

Frequency of the carrier at any instant of time. (ie),

$$\omega_i(t) = \frac{d}{dt}\phi(t)$$

$$= \frac{d}{dt}(\omega_c t + \theta(t))$$

$$\boxed{\omega_i(t) = \omega_c + \theta'(t)} \text{ radians/sec.}$$

(4) Instantaneous Frequency Deviation '$\theta'(t)$':-

It changes in Frequency of carrier at any instant of time. It is defined as first time derivative of instantaneous phase derivation take place.

$$\boxed{\frac{d}{dt}\theta(t) \longrightarrow \theta'(t)} \text{ Hz.}$$

$\theta'(t) \propto V_m(t)$; (ie) $\theta'(t) = K_1 V_m(t)$ radian/sec.

Where, $K_1 \rightarrow$ Deviation sensitivity

(5) Deviation sensitivity: (K)

It is a ratio of output changes to the amplitude of modulating voltage.

For FM,

$$K_{FM} \text{ (or) } K_1 = \frac{\Delta\omega}{V_m} ;$$

For PM,

$$K_{PM} \text{ (or) } K = \frac{\Delta\theta}{V_m} ;$$

PHASE AND FREQUENCY MODULATION:-

* FREQUENCY MODULATION:-

When the Frequency of the carrier varies as per amplitude variation of modulating signal, then it is called Frequency modulation (FM), here amplitude of the modulated carrier remains constant.

* Mathematical Representation of FM:-

Let, modulating signal,

$$V_m(t) = V_m \cos\omega_m t \quad -(1)$$

& carrier signal $V_c(t) = V_c \sin(\omega_c t + \theta)$ —(2)

where,

$V_m \rightarrow$ maximum amplitude of modulating signal

$V_c \rightarrow$ maximum amplitude of carrier signal.

$\omega_m \rightarrow$ Angular Frequency of modulating signal

$\omega_c \rightarrow$ Angular frequency of carrier signal

$\theta \rightarrow$ changes in phase of the carrier.

W.K.T.

$$\phi = \omega_c t + \theta \quad —(3)$$

∴ sub. (3) in (2)

$$V_c(t) = V_c \sin\phi = V_c \sin(\omega_c t + \theta^{(t)}) \quad —(4)$$

Instantaneous frequency (ω_i) is defined as the frequency of the carrier at a given instant of time.

$$\omega_i(t) = \frac{d}{dt}[\omega_c t + \theta(t)]$$

$$\omega_i(t) = \omega_c + \theta'(t) \text{ rad/sec.}$$

In phase modulation (PM); instantaneous phase deviation $\theta(t)$ is proportional to modulating signal voltage.

(ie) $\theta(t) = K \cdot V_m(t)$ rad. ∴ $K \rightarrow$ Deviation sensitivity of Phase.

III^{ly}. In FM, instantaneous frequency deviation $\theta'(t)$ is proportional to modulating signal voltage.

(ie) $\theta'(t) = K_1 V_m(t)$ rad/sec. ∴ $K_1 \rightarrow$ Deviation sensitivity of frequency.

From,

$$\frac{d}{dt}\theta(t) = \theta'(t) \text{ Hz.}$$

$$\theta(t) = \int \theta'(t) \cdot dt$$

$$= \int K_1 V_m(t) \cdot dt$$

$$= K_1 \int V_m \cos\omega_m t \cdot dt$$

$$\boxed{\theta(t) = \frac{K_1 V_m}{\omega_m} \cdot \sin \omega_m t} \quad \leftarrow (5)$$

PHASOR REPRESENTATION

Sub. (5) in (4)

∴ FM equation.

$$V_{FM}(t) = V_c \sin\left[\omega_c t + \frac{K_1 V_m}{\omega_m} \cdot \sin \omega_m t\right]$$

Here, $m_f = \frac{K_1 V_m}{\omega_m}$, (modulating ind of FM)

$$\therefore V_{FM}(t) = V_c \sin[\omega_c t + m_f \sin \omega_m t]$$

lly for PM, Hence FM equation is obtained.

* Mathematical Representation of PM:-

Definition:- When the phase of the carrier varies as per amplitude variation of modulating signal, then it is called Phase modulation (PM). Here, amplitude of modulated carrier remains constant.

$$V_m(t) = V_m \cos \omega_m t \quad -(1)$$

$$V_c(t) = V_c \sin[\omega_c t + \theta(t)] \quad -(2)$$

Phase deviation $\theta(t)$ is proportional to modulating signal voltage. (ie)

$$\theta(t) = K \cdot V_m(t).$$

$$\theta(t) = K_1 \cdot V_m \cos \omega_m t \quad -(3)$$

Sub. (3) in (2), we get.

$$V_{pm}(t) = V_c \sin[\omega_c t + K \cdot V_m \cos \omega_m t]$$

$$m_p = K \cdot V_m$$

$$\therefore V_{pm}(t) = V_c \sin[\omega_c t + m_p \cos \omega_m t]$$

This the equation of Phase modulated wav

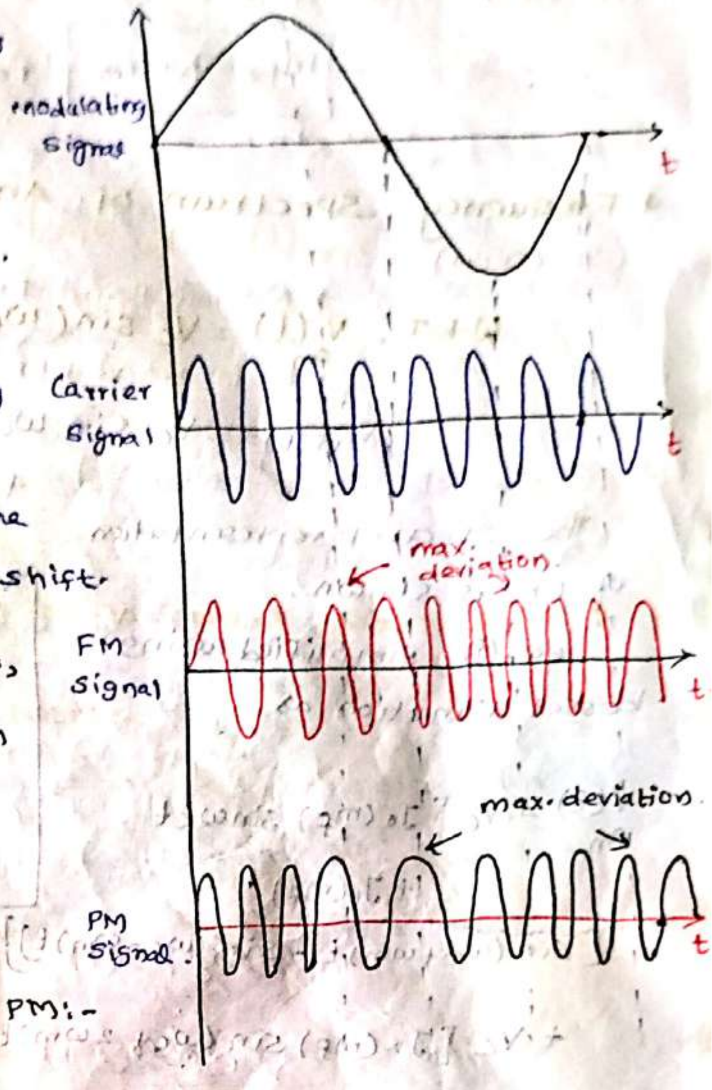
* FM and PM Waveforms :-

✓ For FM, max. deviation takes place when modulating signal is at positive and negative peak.

✓ For PM signal, the max. freq. deviation take place near zero crossing of modulating signal.

✓ Both FM & PM waveform are identical except the phase shift.

(Note: It is difficult to know, whether the modulation is FM or PM from the modulated waveform).



* MODULATION INDEX OF FM & PM :-

FM : (m_f)

Modulation index is directly proportional to peak modulating voltage but inversely proportional to modulating signal frequency.

$$\therefore m_f = \frac{K_1 V_m}{\omega_m} \quad [\because \omega_m = 2\pi f_m]$$

$$m_f = \frac{K_1 V_m}{2\pi f_m} \quad \left[\because \delta = \frac{K_1 V_m}{2\pi}\right] Hz$$

↳ Frequency deviation.

$$\boxed{m_f = \frac{\delta}{f_m}} \text{ unitless}$$

Where,

$\delta \rightarrow$ maximum frequency deviation

$f_m \rightarrow$ modulating frequency.

PM : (m_p):

Modulation index of PM signal is directly proportional to peak modulating voltage. Unit is radians.

$$\boxed{m_p = K \cdot V_m} \text{ radians.}$$

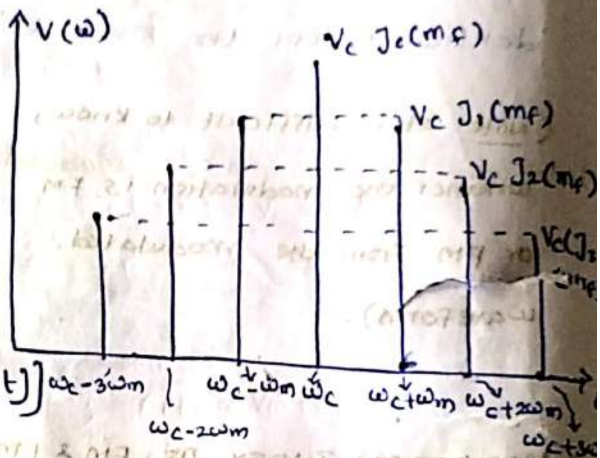
* Frequency Spectrum of Angle modulated wave:-
(SPECTRUM)

W.K.T, $V_{fm}(t) = V_c \sin(\omega_c t + m_f \sin\omega_m t)$

$$V_{fm}(t) = V_c \sin\omega_c t \cos(m_f \sin\omega_m t) + \cos\omega_c t \sin(m_f \sin\omega_m t). \quad \text{—(1)}$$

The spectral representation of FM is shown.

eq. (1), simplified using bessel function as,

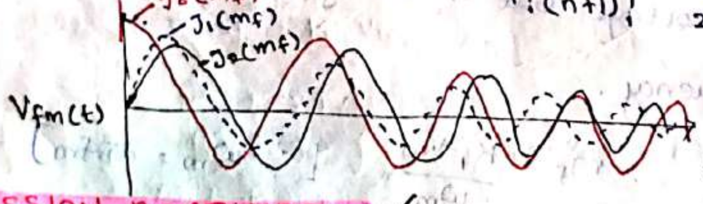


$$V_{fm}(t) = V_c \left[J_0(m_f)\sin\omega_c t + J_1(m_f) \cdot [\sin(\omega_c+\omega_m)t - \sin(\omega_c-\omega_m)t] \right]$$

$$+ V_c \left[J_2(m_f)\sin(\omega_c + 2\omega_m)t - \sin(\omega_c - 2\omega_m)t \right]$$

In general, bessel function,

$$J_n(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n!} + \frac{(m_f/2)^2}{1!(n+1)!} + \frac{(m_f/2)^4}{2!(n+2)!} + \cdots \right]$$



(Plot of bessel function)

* **TRANSMISSION BANDWIDTH:-**

The bandwidth requirement of angle modulated waveforms can be obtained depending up on the modulation index.

The modulation index can be classified as,

- NBFM ← * Low (less than 1) (i.e. $m_f < 1$)
- * Medium (1 to 10) $1 \le m_f \le 10$
- WBFM ← * High (greater than 10) $m_f > 10$

Hence the minimum Bandwidth,

$$\boxed{B.W = 2f_m} \text{ Hz}$$

For High index modulation, the minimum bandwidth is given as,

$$\boxed{B.W = 2\delta} \; ; \quad \delta \rightarrow \text{Maximum Frequency Deviation}$$

The bandwidth can also be obtained using bessel table,

(ie) $\boxed{B.W = 2nf_m}$

where, $n \rightarrow$ Number of significant sidebands obtained from bessel.

(*) 2 mark

* Carson's rule :-

Carson's rule gives approximate minimum bandwidth of angle modulated signal as,

$$\boxed{B.W = 2\left[\delta + f_{m(max)}\right]} \text{ Hz.}$$

* Average Power in FM & PM Modulators :-

The total Power in angle modulated wave is equal to Power of an unmodulated carrier. This mean the Power of an unmodulated carrier is redistributed among the carrier and sideband after modulation.

W.K.T, Avg. Power of carrier,

$$P_c = \frac{V_c^2}{2R}$$

The instantaneous Power in angle modulated carrier can be represented as,

$$P_t = \frac{V^2(t)}{R}$$

$$V(t) = V_c \sin\left[\omega_c t + \theta(t)\right]$$

$$\therefore P_t = \frac{V_c^2}{R}\sin^2\left[\omega_c t + \theta(t)\right] \quad \left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= \frac{V_c^2}{R}\left\{\frac{1}{2} - \frac{1}{2}\cos\left[2\omega_c t + 2\theta(t)\right]\right\}$$

[ ] $\rightarrow$ is cosine term ; Avg. value is zero.

$$\therefore \boxed{P_t = \frac{V_c^2}{2R}}$$

NARROW BAND FM AND WIDE BAND FM:-

The bandwidth of FM signal depends on the modulation index.

If modulation index is high, then B.W is large & vice versa.

FM could be divided in to two types,

* Narrow Band FM ; $m_f < 1$ radian
* Wide Band FM ; $m_f > 1$ radian

NARROW BAND FM:- [NBFM]

Let the message signal be represented as

$$V_m(t) = V_m \cos \omega_m t$$

Carrier signal be given as,

$$V_c(t) = V_c \sin(\omega_c t + \theta) = V_c \sin \phi$$

where, $\phi = \omega_c t + \theta$

After FM,

$$\omega_i = \omega_c + \theta'(t) \quad ; \quad \theta'(t) \propto V_m(t)$$

Note:

$$\omega_i = \omega_c + K_1 V_m \cos \omega_m t \quad ; \quad \theta'(t) = K_1 V_m(t)$$

Frequency deviation is proportional to amplitude of modulating voltage,

$$\therefore 2\pi \Delta f = K_1 \cdot V_m \qquad K_1 = \frac{\Delta \omega}{V_m}$$

$$\therefore \omega_i = \omega_c \pm 2\pi \Delta f \cos \omega_m t$$

$$\phi_i = \int \omega_i dt = \int (\omega_c + 2\pi\Delta f \cos \omega_m t) \cdot dt$$

$$\phi_i = \omega_c t + \frac{\cancel{2\pi}\Delta f}{\cancel{2\pi} f_m} \sin \omega_m t$$

$$\boxed{\phi_i = \omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t}$$

$$\therefore V_{fm}(t) = V_c \sin \phi_i$$

$$= V_c \sin\left(\omega_c t + \frac{\Delta f}{f_m} \cdot \sin \omega_m t\right)$$

$$\boxed{V_{fm}(t) = V_c \sin(\omega_c t \pm m_f \sin \omega_m t)} \quad —(1)$$

W.K.T. already

Expand $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

eq. (1), becomes,

$$V_{fm}(t) = V_c \sin\omega_c t \cdot \cos(m_f \sin\omega_m t) + V_c \cos\omega_c t \sin(m_f \sin\omega_m t)$$

For NBFM, (narrow band)

(x) m_f (modulation index) is small compared to one radian.

Hence, following approximation

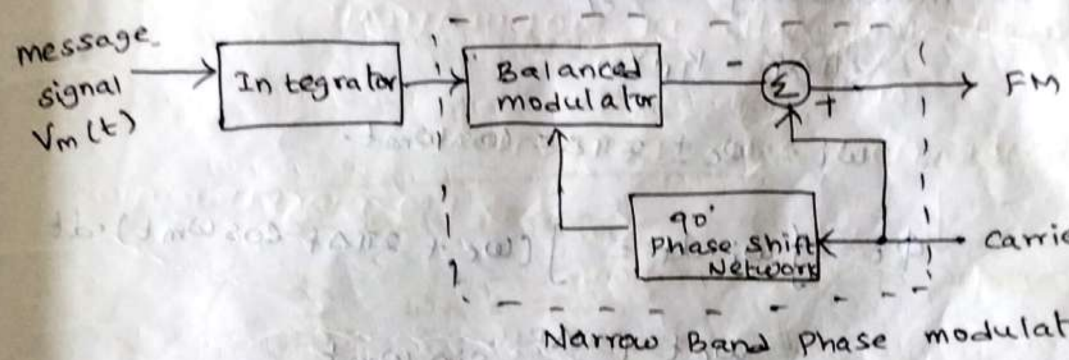
$$\cos(m_f \sin\omega_m t) \approx 1$$

& $\sin(m_f \sin\omega_m t) = m_f \sin\omega_m t$ (Because $\sin(\theta) = \theta$ if 'θ' is small)

$$\therefore V_{fm}(t) = V_c \sin\omega_c t + V_c \cos\omega_c t\,(m_f \sin\omega_m t).$$

↳ This equation defines the narrow band FM. From this, modulator as shown in Figure.

PM to FM



Narrow Band Phase modulator

→ This modulator involves splitting the carrier wave in to two paths. One path is direct, other part contains -90° phase shift network and a product modulator & generate DSB-SC-AM signal.

→ The difference between these two signal produce NBFM with some distortion.

Two condition:-

(1) The envelope contains a residual amplitude modulation and varies with time.

(2) It produce some harmonic distortion.

(It can be avoided by restricting the $m_f \le 0.3$ radians.

WIDE BAND FM:- [WBFM]

In WBFM, the modulation index is large compared to narrow band FM. The WBFM can be obtained by multiplying NBFM signal by using suitable frequency multiplier.

Let the message signal be represented as,

$$V_m(t) = V_m \cos \omega_m t$$

Carrier signal,

$$V_c(t) = V_c \sin(\omega_c t + \theta) = V_c \sin \phi$$

$$\text{where, } \phi = \omega_c t + \theta$$

After FM,

$$\omega_i = \omega_c + K_1 V_m \cos \omega_m t$$

Frequency deviation is proportional to the amplitude of the modulating voltage.

$$2\pi \Delta f = K_1 V_m$$

$$\therefore \omega_i = \omega_c \pm 2\pi \Delta f \cos \omega_m t$$

$$\phi_i = \int \omega_i \, dt = \int (\omega_c + 2\pi \Delta f \cos \omega_m t) \cdot dt$$

$$= \omega_c t + \frac{\Delta f}{f_m} \cdot \sin \omega_m t$$

$$\therefore \boxed{\phi_i = \omega_c t + m_f \sin \omega_m t} \qquad \left[\because m_f = \frac{\Delta f}{f_m}\right]$$

$$\therefore V_{fm}(t) = V_c \sin \phi_i$$

FM signal ;

$$\boxed{V_{fm}(t) = V_c \sin(\omega_c t + m_f \sin \omega_m t)}$$

θ

The above equation is rewritten by using exponential,

$$V_{fm}(t) = V_c \, e^{j(\omega_c t + m_f \sin \omega_m t)}.$$

$$= V_c \, [\cos(\omega_c t + m_f \sin \omega_m t) + j \sin(\omega_c t + m_f \sin \ldots$$

$$V_{fm}(t) = \mathrm{Im}\left[V_c\, e^{j(\omega_c t + m_f \sin\omega_m t)}\right]$$

$$\boxed{V_{fm}(t) = \mathrm{Im}\left[\tilde{V}(t)\cdot e^{j\omega_c t}\right]} \quad \text{— (1)}$$

where,

$$\tilde{V}(t) = V_c\, e^{j m_f \sin\omega_m t} \quad \text{— (2)}$$

The original FM signal $\tilde{V}(t)$, the complex envelope $V(t)$ is periodic function of time with a fundamental frequency equal to modulation frequency f_m.

eq. (2), could be expanded by Fourier Series,

$$V_m(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_m t} \quad \text{— (3)}$$

$C_n \rightarrow$ Fourier co-efficient

$$C_n = f_m \int_{-f_m/2}^{f_m/2} \tilde{V}(t)\cdot e^{-jn\omega_m t}\cdot dt$$

$$C_n = f_m \int_{-f_m/2}^{f_m/2} V_c \cdot e^{j m_f \sin\omega_m t}\cdot e^{-jn\omega_m t}\cdot dt$$

Let,

$$\boxed{\begin{aligned} x &= \omega_m t = 2\pi f_m t \\ \frac{dx}{dt} &= \omega_m = 2\pi f_m \\ \text{(or) } dx &= \omega_m \cdot dt = 2\pi f_m \cdot dt \end{aligned}}$$

limits will change from $-\pi$ to π

$$\therefore \cancel{C_n = V_c} \quad C_n = \frac{f_m}{2\pi f_m}\int_{-f_m/2\ (-\pi)}^{f_m/2\ (\pi)} V_c \cdot e^{j(m_f \sin x - nx)}\cdot dx \qquad \because [\omega_m = 2\pi f_m]$$

$$\therefore C_n = \frac{V_c}{2\pi}\int_{-f_m/2}^{f_m/2} e^{j(m_f \sin x - nx)}\cdot dx. \quad \text{— (4)}$$

The right hand side of integral is n^{th} order bessel function of the first kind.

$$\therefore J_n(m_f) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(m_f \sin x - nx)}\cdot dx.$$

Hence,

$$\boxed{C_n = J_n(m_f) \cdot V_c}$$

$$\tilde{V}(t) = \sum_{n=-\alpha}^{\alpha} J_n(m_f) \cdot V_c \cdot e^{jn\omega_m t} \quad \text{--- (5)}$$

sub. (5) in (1)

$$\therefore V_{fm}(t) = Re\left[\tilde{V}(t) \cdot e^{j\omega_c t}\right]$$

$$= Re\left[\sum_{n=-\alpha}^{\alpha} J_n(m_f) V_c \cdot e^{jn\omega_m t}\right] \cdot e^{j\omega_c t}$$

$$\boxed{V_{fm}(t) = V_c \sum_{n=-\alpha}^{\alpha} J_n(m_f) \cos n(\omega_m t + \omega_c t)}$$

This is the desired form for the fourier series representation of a single tone FM signal for an arbitrary value of m_f.

For WBFM, the m_f is greater than one

VVI

FM MODULATION AND DE-MODULATION:-

(or)

FM GENERATION AND DETECTION:-

→ FM DISCRIMINATORS

Basic principle of FM Generation:-

The frequency of the carrier is varied according to amplitude changes in the modulating signal. The carrier frequency is generated by LC oscillators. The carrier frequency can be changed by varying either inductance or capacitance of tank circuit. Such devices are FET, BJT, varactor diode.

They are two types of FM Generation or FM modulators.

(i) DIRECT FM Generation

(ii) Indirect FM Generation (or) Armstrong method.

FM Generation
- Direct FM
  - FET Reactance Modulator
  - Varactor diode modulator
- Indirect FM (or) Armstrong method FM Generation.

(i) DIRECT FM GENERATION:- ✓

In this type of angle modulation, the Frequency of the carrier is varied directly by the modulating signal. (ie) Instantaneous Frequency deviation is directly proportional to amplitude of the modulating signal.

(a) FET Reactance Modulator

(b) Varactor Diode Modulator.

(a) FET REACTANCE MODULATOR:-

Operation:-

✓ Neglecting gate current, Let the current through 'C' & 'R' be I_1.

✓ At the carrier frequency the reactance of 'C' is much larger than 'R'.

✓ The terminal (A) & (B) of the circuit may be connected across the tuned circuit of oscillator to get FM output.

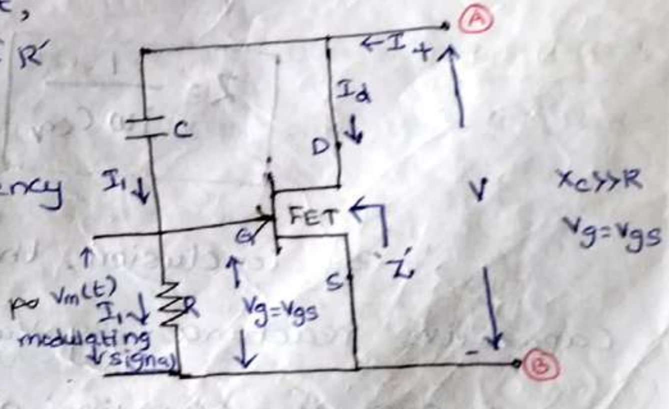


Fig: Circuit diagram of FET Reactance modulator

✓ The varying modulating voltage, across terminal A-B changes reactance of the FET. This changes in reactance can be inductive or capacitive.

The equation for I_1 as,

$$I_1 = \frac{V}{R + \frac{1}{j\omega C}} \quad ; \quad [\text{since, } j\omega C >> R]$$

$$\boxed{I_1 = j\omega CV} \quad \text{—①}$$

From the circuit,

Gate to Source voltage } $V_g = V_{gs} = I_1 \cdot R$ —②

sub. ① in ②

$$\boxed{V_g = j\omega CVR}$$

For the FET,

Drain current } $I_d = g_m \cdot V_{gs}$

$$\boxed{I_d = g_m \cdot j\omega CVR}$$

∴ The impedance of the FET is,

$$Z = \frac{V}{I_d} = \frac{\cancel{V}}{j\omega C\cancel{V}Rg_m} \qquad [\because C_{eq} = g_m \cdot C \cdot R]$$

$$\boxed{Z = \frac{1}{j\omega C_{eq}}}$$

∴ By conclusion, the impedance of the FET is capacitive reactance. By varying modulating signal across FET, the operating point g_m can be varied. Then C_{eq} varies, this changes in capacitance will change the frequency of oscillation.

(b) Varactor Diode Modulator:-

Generally, all the diodes exhibit small junction capacitance in the reverse biased Condition. The varactor diode are specially designed to optimize the characteristics. The junction capacitance of the varactor diode changes as the reverse bias across it is varied. The junction capacitance in the range of 1 to 200 PF.

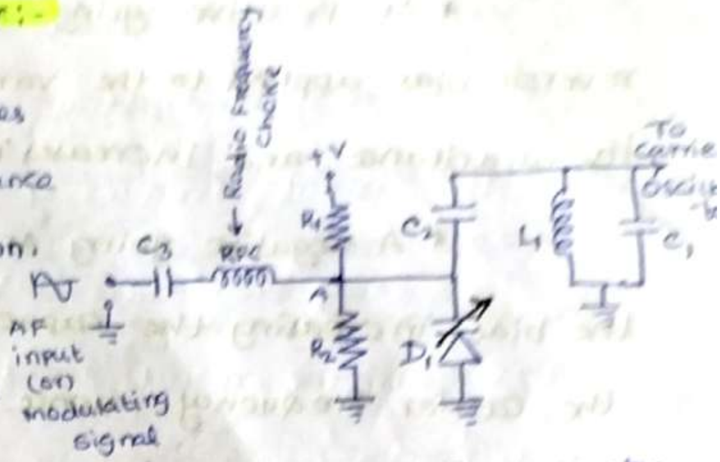


Fig: Circuit for varactor Diode modulator

Operation:-

The Fig. shows the basic concept of a varactor Frequency modulator.

The L1 and C_1 represent the tuned circuit of the carrier oscillator. Varactor diode D_1 is connected in series with the capacitor 'C_2' across the tuned circuit. The total effective circuit capacitance is the capacitance of D_1 is parallel with C_1. This fixes the center frequency.

The capacitance of varactor diode depends up on the fixed bias set by R_1 and R_2 and the AF (Audio Frequency) modulating signal. Either R_1 and R_2 is made variable so that the center carrier frequency can be adjusted over a narrow range.

The RFC has high reactance at the carrier frequency to prevent the carrier signal from getting into the modulating signal circuits.

* At positive going AF signal adds to the reverse bias applied to the varactor diode D_1, which decreases its capacitance and increases in carrier frequency.

* A negative going AF signal subtracts from the bias, increasing the capacitance, which decreases the carrier frequency. Hence FM signal is generated.

Application:-

(i) Automatic Frequency Control (AFC)

(ii) Remote Tuning.

DrawBack in Direct Method of FM Generation:-

✓ It cannot employ crystal oscillator to obtain high frequency stability, so LC oscillator are used to generate NBFM and it is multiplied by appropriate Frequency multiplier network in order to acheive WBFM.

(ii) INDIRECT METHOD OF FM GENERATION:- ✓

(or)

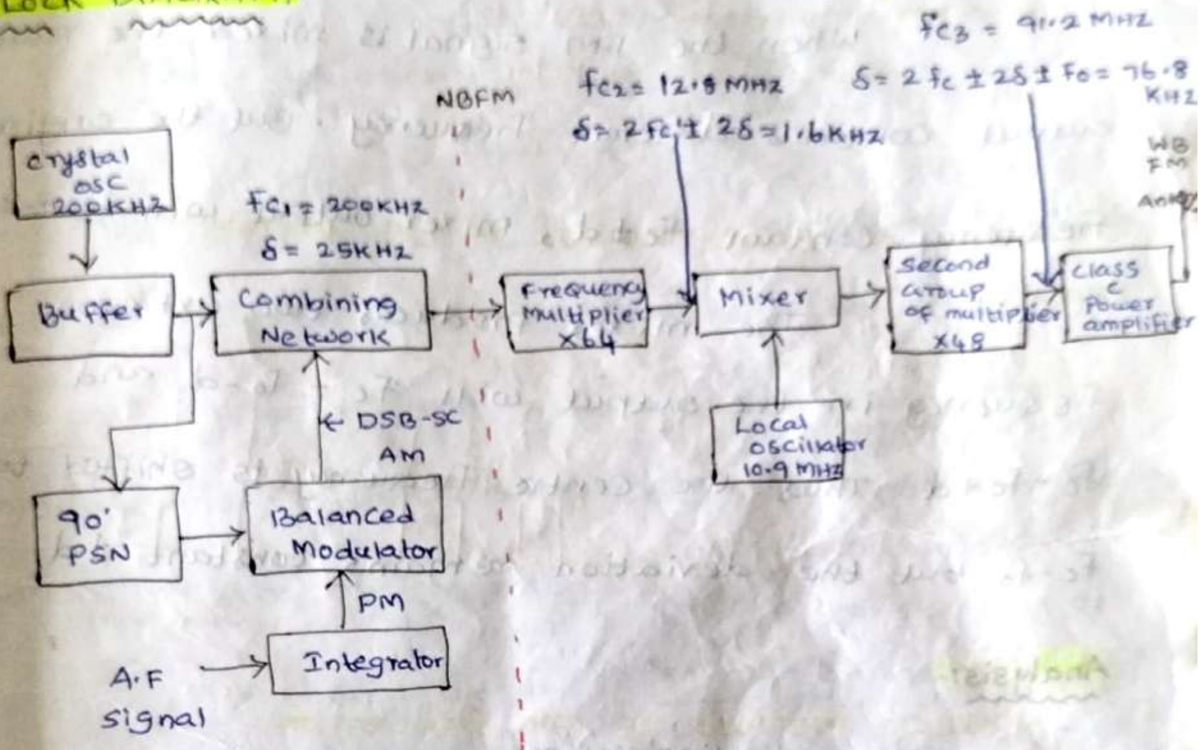
ARMSTRONG METHOD OF FM GENERATION:-

PRINCIPLE:-

In this type FM is obtained by Phase modulation of the carrier. (ie) Instantaneous phase of the carrier is directly proportional to the amplitude of the modulating signal.

In this method a stable crystal oscillator is used to generate PM (phase modulation) from which NBFM is obtained. Then suitable frequency multiplying circuits are used to obtain the desired WBFM. This method is called the Armstrong method of FM wave generation.

BLOCK DIAGRAM:-



(1) In armstrong method of FM wave generation, the cheif advantage is that, the carrier is not directly involving in producing the FM signal, but it is injected and it has no additional frequency corrective circuit is necessary.

The block after the combining network are used to show WBFM might be obtained. The effect of mixer is to change the centre frequency only, where the effect of frequency multiplier is to multiply the f_c and 'δ' equally.

If FM signal ($f_c, \pm\delta$) is fed to frequency doubler or multiplier, the output of signal contains twice the input frequency (ie) ($2f_c, \pm 2\delta$). Thus the frequency deviation has quite clearly doubled to $\pm 2\delta$ with the result modulation index has also be doubled.

When the FM signal is mixed the resulting output contains different frequency. But the original frequency contain $f_c \pm d$, mixer output will be $f_c \pm f_o \pm d$.

The mixer produces two extreme frequency in the output will $f_c - f_o - d$ and $f_c - f_o + d$. Thus the centre frequency is shifted to $f_c - f_o$ but the deviation remains constant $\pm d$.

Analysis:-

Let modulating voltage

$$V_m(t) = V_m \cos\omega_m t \quad -①$$

$$V_c(t) = V_c \sin(\omega_c t + \theta) \text{ if } \theta = 90^\circ \text{ PSN}$$

$$V_c(t) = V_c \sin(90 + \omega_c t) = V_c \cos\omega_c t. \quad -②$$

Audio i/p to Balanced modulator,

$$= \int V_m \cos\omega_m t \cdot dt$$

$$= \frac{V_m}{\omega_m} \sin\omega_m t.$$

Balanced modulator Output,

$$= \frac{V_m V_c}{\omega_m} \cos\omega_c t \sin\omega_m t.$$

$$= \frac{V_m V_c}{2\omega_m}\left[\sin(\omega_c+\omega_m)t + \sin(\omega_c-\omega_m)t\right] \qquad \left[\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}\right]$$

Output of mixer,

$$= V_c \sin\omega_c t + \frac{V_m V_c}{2\omega_m}\left[\sin(\omega_c+\omega_m)t + \sin(\omega_c-\omega_m)t\right]$$

Here,

Peak deviation $\boxed{\phi_p = \tan^{-1}\left(\frac{V_m V_c}{2\omega_m}\right)}$

The mixer output contains a phase deviation ϕ_p and it also contains an AM component. However if the sideband component have very low amplitude, the amplitude modulation is negligible. The frequency deviation produced by the system equals.

$$\Delta f = \phi_p \cdot f_m = \frac{V_m V_c}{2\omega_m} \cdot f_m$$

The above equation shows, the frequency deviation produced by system is directly proportional to magnitude of modulating signal.

Advantage:-

(i) FM is generated from PM indirectly.

(ii) Modulation take place at low carrier frequency unit-II

FM DETECTOR (or) DEMODULATOR:-

FM detector

Frequency Discriminator

(a) Slope detector (or) Single Tuned discriminator

(b) Balanced slope detector (or) Stagger Tuned discriminator (or) Round Travis detector

Phase Discriminator

a) Foster Seely discriminator (or) Centre Tuned discriminator

b) Ratio Detector.

SLOPE DETECTOR:-

PRINCIPLE:-

The original modulating signal is recovered from this AM signal (Converted from FM to AM in previous step) by using a linear diode envelope detector.

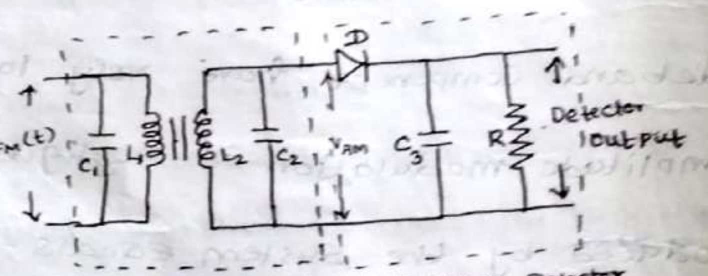


Fig: Slope Detector

The FM discriminator are obtained by using simple RC or LC combination.

Operation:-

✓ The slope detector consists of a Parallel LC tuned circuit which act as a Frequency discriminator. It is slightly detuned from the carrier frequency ω_c.

✓ A low frequency deviation produces small amplitude variation, while a high frequency deviation produces large amplitude variation through this action FM signal is changed to AM signal.

✓ Thus the AM is detected by a diode detector followed by the discriminator circuit.

Advantage:-

(i) It is simplicity in construction

(ii) cheapness.

Drawback:-

(i) The non-linear characteristic of the circuit causes a harmonic distortion.

(ii) It does not eliminate the amplitude variation and the output is sensitive to any amplitude variation in the input FM signal which is not a desirable feature.

BALANCED SLOPE DETECTOR:- [OR] ROUND TRAVIS DETECTOR:-

Triple tuned

Balanced slope detector can be used to overcome the limitation of slope detector (ie) Harmonic distortion is reduced by this detector.

Operation:-

✓ The circuit consists of two LC circuits as shown in fig:a).

✓ The upper portion of secondary winding acts as circuit I and its resonant frequency $f_c + \Delta f$, similarly the lower portion of secondary winding acts as circuit II and its resonant frequency is $f_c - \Delta f$.

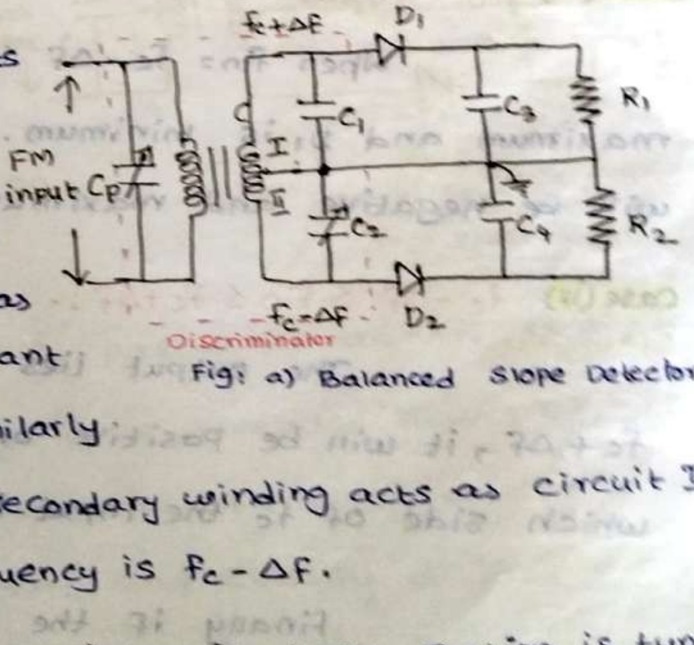


Fig: a) Balanced slope Detector

✓ The output is maximum if upper portion is tuned to $f_c + \Delta f$., above IF (Intermediate Frequency) by an amount Δf and lower portion is tuned below IF, (ie) $f_c - \Delta f$. The output taken across two RC load, when added up gives the total output.

Case(i) $f_{in} = f_c$

When input frequency $f_{in} = f_c$;

the voltage across upper portion will be less than the maximum value. This is because maximum voltage occur at resonance will happen only at $f_c + \Delta f$. The voltage across lower portion will be identical to that of upper part.

∴ output of Diode D_1 is Positive and D_2 is negative, Hence detector output will be zero.

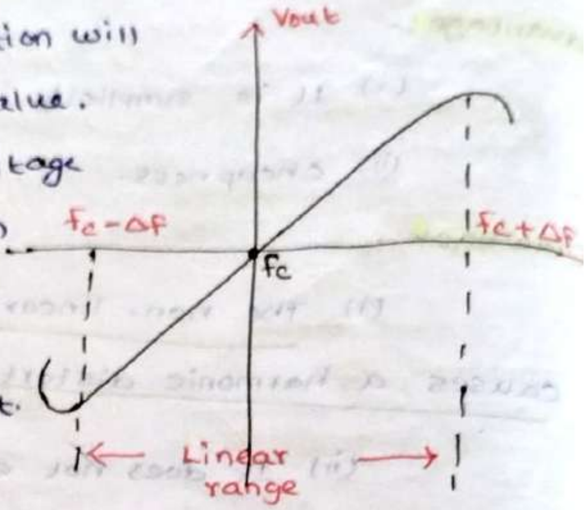


Fig: b) Frequency Response.

[output and Input Frequency characteristics of detector will follow 'S'- shaped curve]

Case(ii) $f_{in} = f_c + \Delta f$

When $f_{in} = f_c + \Delta f$ Circuit I will be at resonance. But in circuit II the input is far away from resonant frequency $f_c - \Delta f$, thus the resultant output will be positive and maximum.

Case(iii) $f_{in} = f_c - \Delta f$

When $f_{in} = f_c - \Delta f$ the output of D_2 will be maximum and D_1 is minimum. Now the resultant output will be negative and maximum.

Case(iv) $f_c - \Delta f \leq f_{in} \leq f_c + \Delta f$:-

The output lies between $f_c - \Delta f$ and $f_c + \Delta f$, it will be positive or negative depending on which side of f_c the input lies.

Finally if the input frequency beyond $f_c + \Delta f$ and $f_c - \Delta f$ the tuned circuit will make the output falls.

Dis-advantages:

(i) Amplitude limiting cannot be provided.

(ii) Linearity is not sufficient when compared with slope detector.

(iii) It is very difficult to align because of three different frequency to which various tuned circuits are to be tuned.

FOSTER SEELY DISCRIMINATOR:-

(OR) CENTRE TUNED DISCRIMINATOR

Principle:

The phase shift between primary and secondary voltages of the tuned transformer is a function of frequency. The secondary voltage lags primary voltage by 90° at the carrier center frequency.

Operation:-

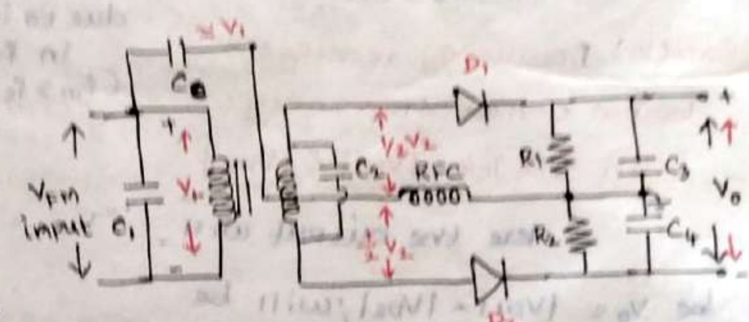


Fig: a) Foster-Seeley Discriminator

In the circuit, primary voltage is coupled through C_c (coupling capacitor) and RFC to the center tap on the secondary.

The capacitor C_c passes all the frequencies of FM. Thus the voltage V_1 is generated across RFC. RFC offers high impedance to frequency of FM. The secondary of voltage is V_2 and equally divided across upper half and lower half of secondary coil.

Fig(b), It observe that the voltage across diode D_1 is $V_{D1} = V_1 + 0.5V_2$ & D_2 is $V_{D2} = V_1 - 0.5V_2$.

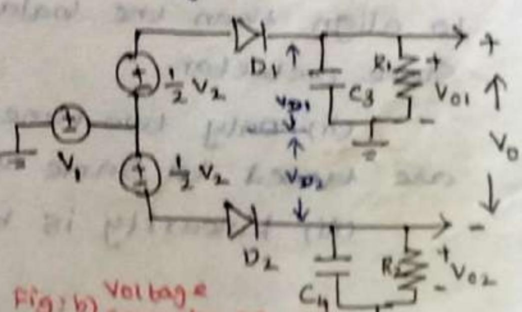
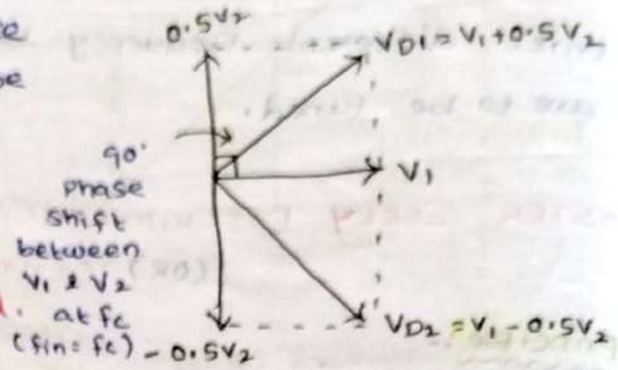


Fig: b) Voltage

The output of upper rectifier is V_{01} and lower rectifier is V_{02}. The output is $V_0 = V_{01} - V_{02}$. (or) $V_0 = |V_{D1}| - |V_{D2}|$

Case (i); At center frequency $|V_{D1}| = |V_{D2}|$ [V_2 will have 90° Phase shift with V_1]

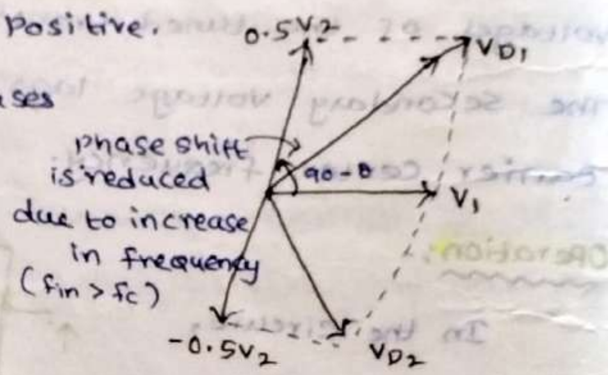
V_{D1} and V_{D2} are generated from V_1 and V_2. In Fig. case(i), vector addition is shown. Hence output of discriminator will be zero.



Case(i) $|V_{D1}| = |V_{D2}|$

Case(ii) Frequency above center frequency [$f_{in} > f_c$] (ie) Phase shift V_1 & V_2 reduced. $|V_{D1}| > |V_{D2}|$:-

Here the output will be $V_0 = |V_{D1}| - |V_{D2}|$, will be Positive. Increases in frequency increases output voltage.

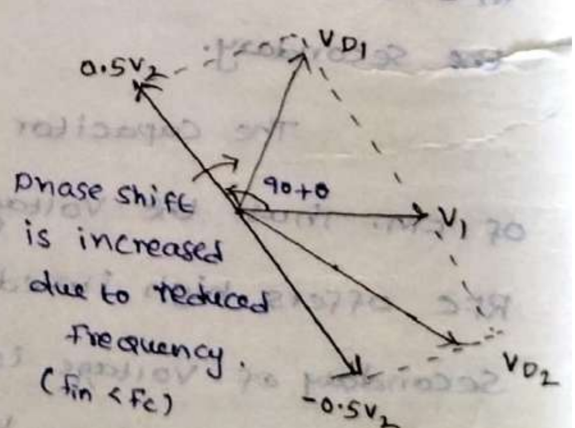


case (ii) $|V_{D1}| > |V_{D2}|$

Case(iii) Frequency reduces below center frequency. [$f_{in} < f_c$] (ie) $|V_{D1}| < |V_{D2}|$

Here the output will be $V_0 = |V_{D1}| - |V_{D2}|$; will be negative.

By conclusion, thus the Foster-Seeley discriminator produces output depends on the Phase shift.



case (iii) $|V_{D1}| < |V_{D2}|$

Advantage :-

(i) It is much easier to align than the balanced slope detector.

(ii) Only two tuned circuit are necessary and both are tuned to same frequency.

(iii) Linearity is better.

Drawback

(i) It needs a seperate amplitude limiting circuit

RATIO DETECTOR:-

Principle:-

Ratio detector can be obtained by slight modification of Foster-Seeley discriminator. Only changes the diode D_2 is reversed, and output is taken from different points.

Operation:-

In Foster Seeley discriminator, the circuit conversion from frequency to phase shift and phase shift to amplitude.

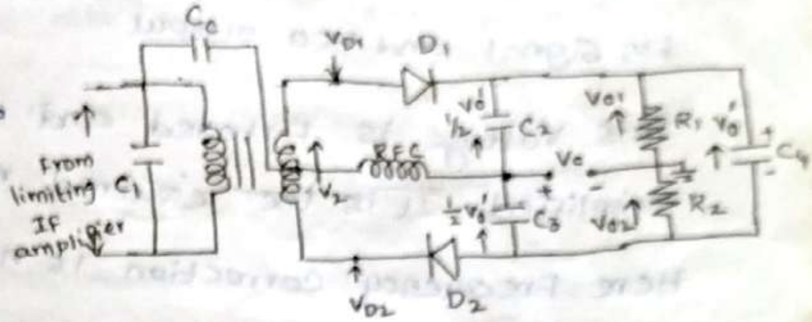


Fig: Ratio Detector

In the ratio detector, the polarity of voltage in the lower capacitor is reversed, since, the connection of diode D_2 are reversed. Hence the voltage across V_{01} and V_{02} across two capacitor C_2 and C_3 add, since V_0' is sum of V_{01} and V_{02}, it remains constant.

From ckt, $\therefore V_0 = \frac{1}{2}V_0' - V_{02}$, &

$$V_0 = -\frac{1}{2}V_0' + V_{01}$$

Diode D_1, $V_0 = \frac{V_{01}-V_{02}}{2}$; D_2, $V_0 = \frac{-V_{02}+V_{01}}{2}$

O/P $2V_0 = \frac{V_{01}-V_{02}}{2} + \frac{-V_{02}+V_{01}}{2}$

$= \frac{1}{2}[V_{01} - V_{02} - V_{02} + V_{01}]$

Add the above two equation,

$2V_0 = \frac{2V_{01} - 2V_{02}}{2}$

$V_0 = \frac{1}{2}[\quad]$

$$2V_0 = V_{01} - V_{02}$$

$$\boxed{V_0 = \frac{1}{2}[V_{01} - V_{02}]} \quad \text{(or)} \quad V_0 = \frac{1}{2}[|V_{D1}| - |V_{D2}|]$$

By conclusion, the output of ratio detector is half compared to that of Foster-Seeley circuit.

Advantage:-

(i) It does not respond to amplitude variation.

(ii) The output is bipolar (ie, positive as well as negative)

Disadvantage:-

(i) It does not tolerate in signal strength over performed period.

(ii) It requires AGC [ ] signal

PLL as FM DEMODULATOR:

The output frequency of VCO is equal to the frequency of unmodulated carrier.

The Phase detector generates the voltage which is proportional to difference between the FM signal and VCO output.

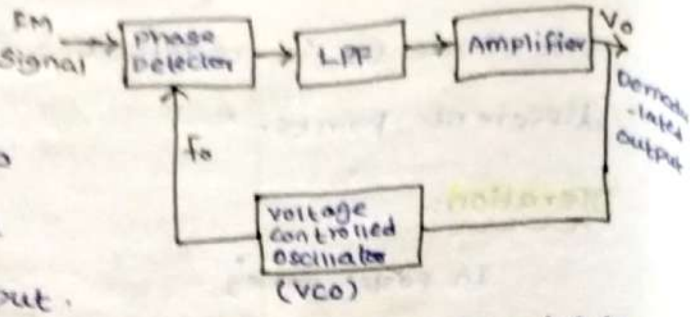


Fig: PLL FM Demodulator

This Voltage is filtered and amplified. It is the required modulating voltage. Here Frequency correction is not required in VCO because it is already done at transmitter.

It consists of

(a) VCO

(b) Phase detector

(c) LPF & Amplifier.

VCO is an electronic oscillator whose oscillation frequency is controlled by a voltage input. The applied input voltage determines the instantaneous oscillation frequency.

Phase detector (or) Phase comparator is a frequency mixer that generate a voltage signal which represent the difference in phase between two signal input (It is an essential element of PLL Phase locked loop). Then it is filtered and amplified to get a desired modulating signal.

Advantage:

(i) No need of Tuned Circuit

(ii) Simple circuit that can be implemented in integrated circuits.

Comparison of AM (Amplitude Modulation) and Frequency Modulation:-

AM	FM
(i) Amplitude of carrier is varied according to amplitude of modulating signal.	(i) Frequency of carrier is varied according to amplitude of modulating signal.
(ii) AM has poor fidelity due to narrow Bandwidth	(ii) Fidelity is Better, since Bandwidth is large.
(iii) Most of the power is in carrier hence less efficient.	(iii) All the transmitted power is useful.
(iv) Noise interference is more	(iv) Noise interference is minimum.
(v) Adjacent channel interference is present.	(v) Adjacent channel interference is avoided due to large Bandwidth.
(vi) AM broadcast operate in MF (medium frequency) and HF (High Frequency) range.	(vi) FM broadcast operate in VHF and UHF range.
(vii) AM has only carrier and two side bands are present.	(vii) Infinite number of side bands.
(viii) Transmission equipment is simple.	(viii) It is complex
(ix) AM is used for transmission of sound, video in TV.	(ix) FM is used for audio broadcasting and mobile, Base stations.

Problems:-

1) An angle modulated wave is described by the equation $V(t) = 10\cos(2\times10^6\pi t + 10\cos 2000\pi t)$. Find (1) Power of modulated signal (2) Maximum frequency deviation (3) Bandwidth.

Solution:-

$$V(t) = 10\cos(2\times10^6\pi t + 10\cos 2000\pi t) \quad \text{— ①}$$

W.K.T, FM signal

$$V_{fm}(t) = V_c \sin(\omega_c t + m_f \sin \omega_m t) \quad \text{— ②}$$

compare ① & ②, we get

$A_c = 10$

$w_c = 2\times10^6\pi$

$w_m = 2000\pi$

$V_c = 10$ volts

$\omega_c = 2\times10^6\pi$

$m_f = 10$

$\omega_m = 2000\cancel{\pi} = 2\cancel{\pi} f_m$

$f_m = \frac{\overset{1000}{\cancel{2000}}}{\cancel{2}}$

$\boxed{f_m = 1000}$ Hz

(1) Power of modulated signal:-

$P_t = \frac{V_c^2}{2R} = \frac{(10)^2}{2\times1}$ [For normalized Assume $R=1$]

$\boxed{P_t = 50}$ watts

(2) Maximum frequency deviation:- (δ)

"δ" in terms of modulation index is,

W.K.T

$\delta = m_f \cdot f_m$

$m_f = \delta / f_m$

$\delta = 10\times1000$

$\boxed{\delta = 10{,}000}$ Hz

(3) Bandwidth:-

Carson rule $\} \rightarrow$ $B.W = 2[\delta + f_{m(max)}]$

$= 2[10{,}000 + 1000]$

$\boxed{B.W = 22\text{ KHz}}$

$(SNR)_i$:- Signal to Noise Ratio at the input :-

It is defined as the ratio of signal Power at input (P_{si}) to the Noise Power at the input (P_{ni})

$$\therefore (SNR)_i = \frac{\text{Signal power at input}}{\text{Noise Power at input}}$$

$$\boxed{(SNR)_i = \frac{P_{si}}{P_{ni}}}$$

$(SNR)_o$:- Signal to Noise Ratio at the Output:-

It is defined as the ratio of signal Power at output (P_{so}) to the Noise power at output (P_{no}).

$$\therefore \boxed{(SNR)_o = \frac{P_{so}}{P_{no}}}$$

$\therefore$ EQ. (1) becomes,

$$\boxed{F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}} \quad \text{--- (2)}$$

In many cases, the noise factor 'F' is always unity depends on frequency and it is calculated at one signal frequency it is known as Spot noise Factor.

Power Gain 'G' of the circuit is,

$$G = \frac{\text{Signal power at output}}{\text{Signal power at input}} = \frac{P_{so}}{P_{si}}$$

$\therefore$ eq. (2) becomes,

$$\cancel{F = \frac{P_{si}}{P_{ni}} \times \frac{1}{G}}$$

P.

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}} \qquad \left[\because G = \frac{P_{so}}{P_{si}}\right]$$

$$F = \frac{P_{no}}{P_{ni}} \times \frac{1}{G}$$

$P_{no} = P_{ni} \times F \times G$

$\boxed{P_{no} = F G P_{ni}}$ —③ W.K.T, Noise power at input (Thermal noise)

$\boxed{P_{ni} = KTB}$

∴ eq③ becomes,

$\boxed{P_{no} = FGKTB}$ —④

Where,

- $F \rightarrow$ Noise factor
- $G \rightarrow$ Gain in the amplifier
- $K \rightarrow$ Boltzmann constant
- $T \rightarrow$ Temperature in °K
- $B \rightarrow$ Bandwidth in Hz

NOISE FIGURE:-

Noise factor is expressed in decibel, it is known as "noise figure".

$$\boxed{\text{Noise Figure} = 10 \log [F]}$$

W.K.T, from eq.④

$P_{no} = FGKTB$

∴ Total noise power at input

$P_{ni} = KTB$

From③, $P_{ni} = \frac{P_{no}}{G} = \frac{FKTB\cancel{G}}{\cancel{G}}$

$\boxed{P_{ni} = FKTB}$

Source contributes 'KTB' and hence the amplifier contributes noise power, P_{na} gives by,

P_{na} = Total P_{ni} - P_n due to source.

P_{na} = FKTB - KTB

$$\boxed{P_{na} = (F-1)KTB}$$

The Fraction of total available noise contributed by the amplifier,

$$= \frac{(F-1)KTB}{FKTB} = \frac{(F-1)}{F}$$

NOISE TEMPERATURE :-

DEFINITION:-

The available noise power is directly proportional to temperature and it is independent of value of resistance. This Power specified in terms of temperature is called noise temperature.

W.K.T

Noise Power due to amplifier, having a noise factor 'F' is

$$\boxed{P_{na} = (F-1)KTB} \quad \text{--- (1)}$$

If T_e → Equivalent noise temperature represent noise power,

From Definition,

$$\boxed{P_{na} = KT_eB} \quad \text{--- (2)}$$

To determine Noise temperature T_e, equating (1) = (2)

$$\cancel{K}T_e\cancel{B} = (F-1)\cancel{K}T\cancel{B}$$

$$\boxed{T_e = (F-1)T} \quad \text{--- (3)}$$

Using Friss Formula,

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots \quad \text{--- (4)}$$

Subtract (1) on both side in eq. no. (4)

$$F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots \quad \text{--- (5)}$$

From the noise Temperature,

$$T_e = (F-1) T$$

$$(F-1) = \frac{T_e}{T}$$

Sub $(F-1) = T_e/T$ in eq. no. (5)

$$\therefore \frac{T_e}{T} = \frac{T_{e1}}{T} + \frac{\left(\frac{T_{e2}}{T}\right)}{G_1} + \frac{\left(\frac{T_{e3}}{T}\right)}{G_1 G_2} + \cdots$$

$$\boxed{T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots}$$

NOISE IN CASCADE SYSTEM:- [OR] "FRISS'S FORMULA"

Consider two amplifier connected in cascade. For one amplifier, the power gain is G_1 and its noise factor F_1; While the corresponding figures for the other amplifier are G_2 and F_2, respectively. The Figure shown below

When the amplifier consists of more than one stage its noise figure can be computed in terms of noise figure of individuals stages.

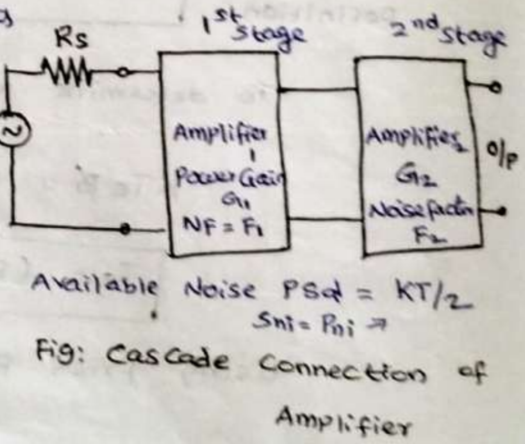


Fig: Cascade Connection of Amplifier

First determine (S_{no}) ie, Available total noise power at output terminals,

S_{no} consists of two components,

$S_1 \rightarrow$ Total noise power density available at the output due to 1^{st} stage noise.

$S_2 \rightarrow$ Total noise power density available at the output due to 2^{nd} stage noise.

Noise psd (power spectral density) from any two terminal network can be written as,

$$\boxed{S_{ni} = \frac{KT}{2} = P_{ni}} \quad -①$$

W.K.T, Noise factor (F), from eq. ① in noise factor

$$\boxed{F = \frac{P_{no}}{G P_{ni}}} \quad -②$$

sub ① in ② $\Rightarrow$ becomes,

$$F = \frac{P_{no}}{G(KT/2)} = \frac{2P_{no}}{GKT}$$

$$\therefore \boxed{P_{no} = \frac{FGKT}{2}}$$

Then psd at output due to 1^{st} stage of amplifier is written as,

$$\boxed{S_1 = \frac{F_1 G_1 KT}{2}}$$

Let the noise figure of amplifier 2 is F_2. The noise psd at the output due to 2^{nd} stage of amplifier S_2 is given as,

$$\therefore \boxed{S_2 = \frac{F_2 G_2 KT}{2} - \frac{G_2 KT}{2}}$$

1st term → $\frac{F_2 G_2 KT}{2}$ → Total noise Power density at output of 2nd stage.

2nd term → $\frac{G_2 KT}{2}$ → Noise Power density due to Second amplifier.

$$\therefore \boxed{S_2 = \frac{G_2 KT}{2} (F_2 - 1)}$$

and S_{21} → noise Power due to amplifier 1 which is amplified by amplifier '2'.

$$\boxed{S_{21} = \frac{F_1 KT G_1 G_2}{2}}$$

Hence, Total noise power at output

$$(S_{no})_{av} = S_{21} + S_2$$

$$(S_{no})_{av} = \frac{G_2 KT}{2} (F_2 - 1) + \frac{F_1 KT G_1 G_2}{2}$$

$$\boxed{(S_{no})_{av} = \frac{KT}{2} \left[(F_2 - 1) G_2 + F_1 G_1 G_2 \right]}$$

Where,

$$G_{ab} = G_1 G_2$$

$$F_{ab} = F_1 + \frac{F_2 - 1}{G_1}$$

$$\boxed{(SN_o)_{av} = \frac{KT}{2} G_1 G_2 \left[F_1 + \frac{(F_2 - 1)}{G_1} \right]} \quad \text{--- (3)}$$

For, multistage amplifier, the noise factor is given as,

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots$$

From above equation, noise factor 'F' is depends on gain of the amplifier stage. It is known as "Friss's Formula".

NARROW BAND NOISE - PSD OF INPHASE AND QUADRATURE NOISE :-

Definition:-

The receiver of a communication system usually includes some provision for preprocessing the received signal. The preprocessing ~~the~~ may take the form of a narrow band filter whose bandwidth is just large enough to pass the modulated component of a received signal.

The noise appearing at the output of such a filter is called narrow band noise. The spectral component of narrow band noise concentrated about some mid-band frequency $\pm f_c$.

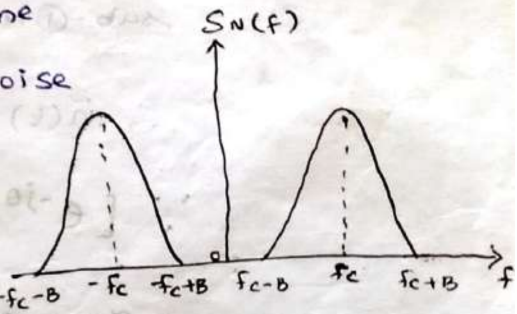


Fig:- PSD of Narrow Band Noise.

To analyze the effect of narrow band noise on the performance of a communication system, we need a mathematical representation of it. Depending on the application of interest, there are two specific representation of NBN.

(1) In terms of a pair of component called inphase and quadrature component.

(2) Represented in terms of Envelope and Phase.

The sample function of such narrow band noise $n(t)$ appears as a sine wave of frequency f_c which modulates slowly in amplitude and phase.

$$\boxed{S_N(f) = \frac{N_0}{2} |H(f)|^2}$$

Representation of Narrow Band noise in terms of inphase and Quadrature Components:-

Generally, $n(t)$ can be represented by its pre-envelope and complex envelope as follows,

Pre envelope and Complex envelope in terms of signal $\Big\} \Rightarrow$

$$x_p(t) = x(t) + j\hat{x}(t)$$

$$x_c(t) = x_p(t) \cdot e^{-j2\pi f_c t}$$

Similarly in terms of noise, pre-envelope and complex envelope is represented as,

Pre envelope $n_+(t) = n(t) + j\hat{n}(t)$ — ①

Complex $\tilde{n}(t) = n_+(t) \cdot e^{-j2\pi f_c t}$. — ②

sub. ① in ②

$$\therefore \tilde{n}(t) = [n(t) + j\hat{n}(t)] \cdot e^{-j2\pi f_c t}$$

$$\because [e^{-j\theta} = \cos\theta - j\sin\theta]$$

$$\therefore \tilde{n}(t) = [n(t) + j\hat{n}(t)][\cos 2\pi f_c t - j\sin 2\pi f_c t]$$

$$= n(t)\cos 2\pi f_c t - j\, n(t)\sin 2\pi f_c t + j\hat{n}(t)\cos 2\pi f_c t + \hat{n}(t)\sin 2\pi f_c t$$

$$\boxed{\tilde{n}(t) = n_c(t) + j n_s(t)}$$

where,

real Inphase Component of NBN $\Big\} \rightarrow n_c(t) = n(t)\cos 2\pi f_c t + \hat{n}(t)\sin 2\pi f_c t$

imaginary Quadrature Component of NBN $\Big\} \rightarrow n_s(t) = \hat{n}(t)\cos 2\pi f_c t - n(t)\sin 2\pi f_c t$

Hilbert transform:

It defines the correlation between the bandpass signal and system and its equivalent baseband signal and system.

Hilbert transform does not change the domain of the signal. The HT of the signal $g(t)$,

$$\boxed{\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau.}$$

It is represented by the above eqn.

Random Variable :

A function that maps every outcomes in a sample space to a real number is called as a random variable.

Random Variables are denoted by X, Y and the values taken by them are represented by x_1, x_2, y_1, y_2 etc. Random variables may be classified into.

i) Discrete Random Variable

ii) Continuous Random Variable.

Discrete probability distribution:

Let X be a discrete random variable and let $x_1, x_2, x_3 \ldots$ are the values of X, then

$P(X = x_j) = f(x_j)$, $j = 1, 2, 3 \ldots$ (1).

$f(x_j)$ is the probability of x_j.

$f(x_j)$ (or) $f(x)$ - probability function.

Properties :-

i) $f(x) \geq 0$ and

ii) $\sum_{x} f(x) = 1$.

ii) Cumulative distribution function (CDF):

The CDF of a random variable 'x' defined as the probability that a random variable x takes a value less than or equal to x.

ie CDF: $F_X(x) = P(X \leq x)$.

properties:

i) $F_X(\infty) = 1$

ii) $F_X(-\infty) = 0$.

iii) $0 \leq F_X(x) \leq 1$.

iii) Probability density function (PDF).

The derivative of CDF with respect to some dummy variable 'x' is known as PDF.

$$f_X(x) = \frac{d}{dx} F_X(x).$$

Central Limit Theorem:

The central limit theorem states that the probability density of a sum of 'N' independent random variable tends to approach a normal density as the number 'N' increases.

Let x_i - 1, 2, 3, 4 ... N be a set of random variables that satisfies the following requirements

i) The x_i are statistically independent.
ii) The x_i have the same probability

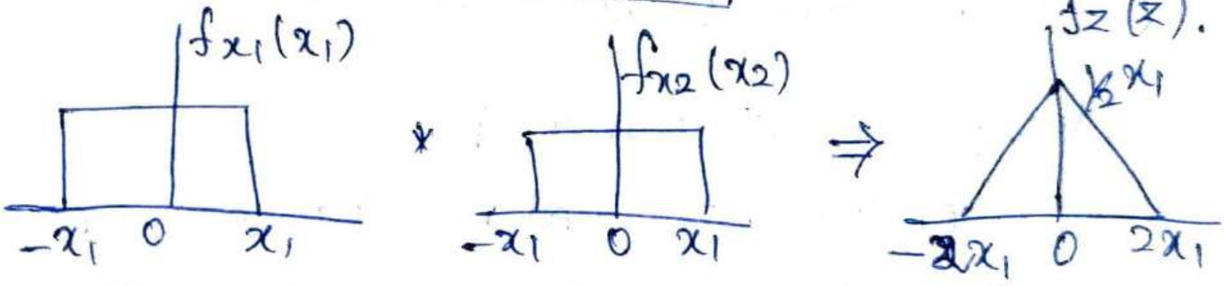
$$y_i = \frac{1}{\sigma_x}(x_i - \mu_x) \quad i = 1, 2 \ldots N$$

So that we have,

$$E[y_i] = 0 \text{ and } var[y_i] = 1$$

Then the random variable as.

$$\boxed{V_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} y_i}$$



PDF of RV X_1 PDF of RV X_2 PDF of $Z = x_1 +$

<u>Properties</u> <u>of Central limit theorem</u>:

1. the mean of Gaussian PDF obtained by addition of N independent random variable is equal to the sum of means of the random variable being added.

2. The variance of Gaussian PDF is equal to sum of variances of N independent random

variables which are being added.

3. The central limit theorem applies even when the individual random variables are not Gaussian.

Random Process:

The ensemble comprised of function of time is called as random process or stochastic process. Consider a random experiment specified by the outcomes from some sample space S by the events and probabilities of these events in sample space.

Each sample point $= X(t, s)$, $-T \leq t \leq T$.

Classification of Random process:

i) Stationary and non stationary R.P.
ii) wide sense (or) weakly stationary process.
iii) Ergodic process.

i) Stationary Random Process:

A random process whose statistical characteristics do not change with time. Hence shift of time origin will not have any effect on the stationary R.P

Non stationary Random process:-

A random process whose statistical characteristics will change with time.

ii) <u>wide sense stationary RP:</u>

A process may have mean value $m_x(t)$ and autocorrelation function which are independent of the shift of time origin.

i.e $R_x(t_1, t_2) = R_x(t_2 - t_1)$.

such a process is known as WSS. All stationary process are WSS but every WSS may not be strictly stationary.

iii) <u>Ergodic process:</u>

The time average $m_x(T)$ approach the ensemble average m_x with the observation interval T tending to infinity.

$$\lim_{T \to \infty} \underbrace{m_x(T)}_{\text{Time average}} = \underbrace{m_x}_{\text{ensemble average}}.$$

Stationary process:-

The random process $x(t)$ is said to be stationary in the strict sense if the joint CDF of the original set of R.V is equal to that of new set of random variables

Obtained after time shift τ.

Property :

1. The first order distribution function of Stationary random process is independent of time.

2. The 2nd order distribution function of S.R.P depends only on time difference between the Observation times and not depend on particular time at which random process is observed.

Mean, correlation and Covariance functions :-

Mean :

Consider a random process $x(t)$ assumed to be strictly stationary. Let $x(t_k)$ denote the random variable obtained by observing the process $x(t)$ at time t_k.

The mean of process is

$$m_x(t) = \text{expected value of } x(t).$$

$$m_x(t) = m_x = E[x(t_k)] = \int_{-\infty}^{\infty} x f_{x(t)}(x)\, dx.$$

where $f_{x(t)}(x)$ is 1st order PDF.

Correlation :

The autocorrelation function of stationary process $x(t)$ is

$R_X(t_k - t_j) = E[x(t_k)\, x(t_j)]$ for any t_k and t_j.

$R_X(\tau) = E[X(t) \cdot X(t-\tau)]$.

$R_X(t_1, t_2) = E[X(t_1)\, x(t_2)]$.

$$R_X(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2\, f_{X(t_1)} f_{X(t_2)}(x_1, x_2)\, dx_1 dx_2.$$

The autocorrelation function $R_X(\tau)$ is independent of shift of time.

Covariance:

The autocovariance function of a stationary process is given by

$$C_X(t_2 - t_1) = E[(X(t_1) - m_x)(x(t_2) - m_x)] = R_X(t_2 - t_1) - m_x^2.$$

It is possible to calulate autocovariance if mean and autocorrelation of random process are known.

Power spectral density:-

It is used to analyse the characteristics of random process in linear systems by using frequency domain.

$$h(\tau_1) = \int_{-\infty}^{\infty} H(f) \cdot e^{(j2\pi f \tau_1)} \cdot df \quad -①$$

The mean square value of o/p process is

$$E[y^2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1) h(\tau_2) R_X(\tau_2 - \tau_1)\, d\tau_1 d\tau_2 \quad -②$$

Substitute ① in ② we get.

$$E[y^2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} H(f)\, e^{(j2\pi f \tau_1)} df\right] h(\tau_2)$$
$$R_X(\tau_2 - \tau_1)\, d\tau_1 d\tau_2$$

Rearranging and after simplification,

$$E[y^2(t)] = \int_{-\infty}^{\infty} H(f) H^*(f)\, df \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau$$

$$= \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau.$$

i.e $x(t) \overset{F}{\longleftrightarrow} \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau.$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau.$$

↳ power spectral density.

Ergodicity in autocorrelation:

The necessary and sufficient condition for ergodicity of autocorrelation is that variance of estimator approach zero as T approaches infinity.

Gaussian process:

Consider a random variable y with random process $x(t)$ with interval 0 to T. Then random variable y is given by.

$$y = \int_0^T g(t)\, x(t)\, dt \quad \text{--- (1)}$$

where y is a linear function of $x(t)$.

There is a difference between linear function and linear functional. for example,

$y = \sum_{i=1}^{N} a_i x_i \Rightarrow y$ is linear fn. of x_i.

$y = \int_0^T g(t) x(t) dt \Rightarrow y$ is linear fn'al of x_i.

because y depends on argument function $g(t)\, x(t)$.

The $g(t)$ in equation ① represents mean square value of random variable y is finite if the random variable y is Gaussian distributed random variable for every $g(t)$, then the process $x(t)$ is said to be Gaussian process.

The PDF of Gaussian is

$$f_{x(t_i)}(x_i) = \frac{1}{\sigma_{(t_i)}\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\frac{x_i - m(t_i)}{\sigma(t_i)}\right]^2}$$

Transmission of random process through LTI filter :-

Consider a random process $x(t)$ which is WSS, is applied as input to linear time invariant filter of impulse response $h(t)$, producing a random process $y(t)$ at the filter output.

$$\xrightarrow[x(t)]{i/p} \boxed{\text{Impulse response } h(t)} \xrightarrow{} o/p\ y(t)$$

To determine the mean and autocorrelation function of the output random process $y(t)$ in terms of $x(t)$

$$m_y = m_x \int_{-\infty}^{\infty} h(\tau)\, d\tau.$$

Noise Sources and Types – Noise Figure and Noise temperature – Noise in Cascade systems – Narrow band noise – PSD of in-phase and Quadrature noise – Noise Performance in AM Systems – Noise Performance in FM Systems – Pre-emphasis and de-emphasis – Capture Effect, Threshold effect.

NOISE:-

Noise is defined as unwanted signal that tends to interfere with the required signal. Because of noise, the transmission of signal is disturbed and errors are introduced in the received signal.

The mathematical representation of noise is very important and helps to analyzing the effect of noise on the performance of system.

NOISE SOURCES AND ITS TYPES:-

The classification of noise is shows,

NOISES

- EXTERNAL NOISE
  - NATURAL NOISE
    - (a) Atmospheric noise
    - (b) Extra Terrestrial noise.
      - Solar noise
      - cosmic noise
  - MANMADE NOISE
    - Industrial (c) noise.
- INTERNAL NOISE
  - (i) Thermal noise (or) Thermal agitation (or) Johnson noise.
  - (ii) Shot noise
  - (iii) Partition noise
  - (iv) Low Frequency (or) Flickering noise.
  - (v) High Frequency (or) Transit time

EXTERNAL NOISE :-

a) Atmospheric Noise :-

The noise is produced due to several electrical disturbances, such as lightening dischagges in thunderstroms. These disturbance are also called as Static noise. The electrical dischagges that occur between clouds or between the earth and clouds. It greatly affect the reception at frequency less than 30MHZ.

b) Extraterrestrial Noise:-

Extraterrestrial noise comes from Sources in space, which are again divided in to two groups:

(i) Solar noise

(ii) Cosmic noise

(i) Solar noise:-

Primary source of solar noise is sun. The sun radiates a wide range of signal in a broad noise spectrum which include the frequencies we use for communication. These noise radiation produced by sun, vary with time. The Sun cycle repeats after 11 years.

(ii) Cosmic noise:-

It is generated by stars, which also suns. The level is not as great because of the great distance between the stars and earth. It has a greatest impact on communication in the frequency range from 15MHZ to 150MHZ.

(c) Industrial Noise (Man-Made noise):-

It is produced by automotive ignition systems, electrical motors and generator brush contacts.

An electrical equipment when abruptly switched 'ON' or 'OFF' produces 'transients' that create noise. Florescent light also produce noise. The way to control it is to use high powered signal for transmission which will increase signal level at receiving point.

INTERNAL NOISE:-

Noise generated internally in the circuit. Electronic components such as resistors, diodes and transistor produce noise (low level noise), it can interfere with weak signal. By means of proper design, the effect of noise are minimized.

* Noise Power across resistor is directly Proportional to the Temperature.

(i) Thermal noise (or) Agitation (or) Johnson noise:-

The electron in a conductor Possess varying amount of energy by virtue of temperature of conductor. The small fluctuation in energy are sufficient to produce small noise voltage in the conductor. These random fluctuation produced by thermal agitation of the electrons are called thermal noise.

The average noise power is proportional to bandwidth and absolute Temperature of the conductor.

$P_n \propto TB$

$$\boxed{P_n = KTB} \quad \text{—(1)}$$

watts

where,

$P_n \rightarrow$ Avg. power in noise

$T \rightarrow$ Temperature of conductor ($^\circ K$)

$B \rightarrow$ Bandwidth

$K \rightarrow$ Boltzmann's Contant $= 1.38 \times 10^{-23}$ J/K

The power spectrum density is the average noise power per hertz of bandwidth, Then

$$S_n = \frac{P_n}{B} = \frac{KT\cancel{B}}{\cancel{B}}$$

$$\boxed{S_n = KT} \text{ watts}/\text{Hz}$$

Consider R_L is noiseless resistor, receiving the maximum noise power generated by 'R'

as per the condition of maximum Power transfer theorem,

$R = R_L$

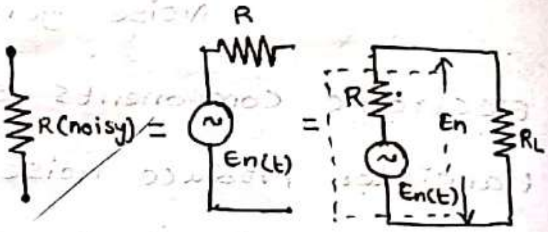


Fig: Thevenin's Equivalent ckt for noise source.

W.K.T,

Noise Power, $P_n = \frac{V^2_{rms}}{(R_L+R)} = \frac{E^2_{rms}}{2R} = \frac{(E_n/\sqrt{2})^2}{2R}$ $\quad \frac{Amp}{\sqrt{2}}$

$$\boxed{P_n = \frac{E_n^2}{4R}}$$

$\therefore E_n^2 = 4RP_n$ $\quad$ From eq.① $\left[\because \text{W.K.T } P_n = KTB\right]$

$E_n^2 = 4RKTB$

$$\boxed{E_n = \sqrt{4KTBR}}$$

$E_n \rightarrow$ Peak or max. value of noise voltage

If two resistor are connected in Series, then,

$$\boxed{E_n = \sqrt{4KTB(R_1+R_2)}}$$

If two resistor are Parallel, then,

$$\boxed{E_n = \sqrt{4KTB\left(\frac{R_1R_2}{R_1+R_2}\right)}}$$

(ii) Shot noise :-

Shot noise is present in both vaccum tube and semiconductor devices. In vaccum tube shot noise occur due to random emission of electron from the cathode. In semiconductor devices such as diode, transistor, this effect arises due to random diffusion of minority carrier as well as random generation and recombination of electron hole pairs.

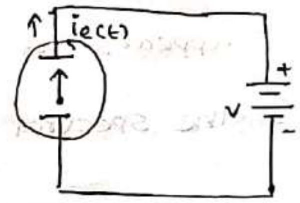


Fig: Shot noise Source.

Psd of shot noise :-

power spectrum density

Shot noise is gaussian distributed with zero mean.

$$\boxed{S(\omega) = q I_0}$$

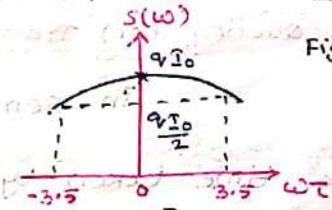


Fig: PSD of shot noise

where, $q \rightarrow 1.5 \times 10^{-19}$ Colombs

$I_0 \rightarrow$ mean value of current in Amperes.

(iii) Partition Noise :-

This occur whenever current has to divide between two or more electrodes results in random fluctuation in the current division.

It is very less in diodes than transistor because ~~in transistor~~ the third electrode draws current.

eg: BJT, random motion of carrier crossing (Emitter - Base), (Base - collector) and random recombination of electron and holes in the base.

Mean squared value of Partition noise in transistor is,

$$I_{ne} = \sqrt{2 I_e \left(1 - \frac{|\alpha|^2}{\alpha_0}\right)}$$

where,

I_e - Avg. collector current

$\alpha \rightarrow$ current amplification factor

$\alpha_0 \rightarrow$ current amplification at low frequencies

(iv) Low Frequency (or) Flicker Noise :-

At low frequencies (below few KHz), a component of noise appears, the spectral density of which increases as the frequency decreases. This is known as Flicker noise.

Burst noise:-

It is another type of low frequency noise occurs in transistor the name arising because the noise appears as a series of burst at two or more levels. The spectral density increases as frequency decreases.

(V) High Frequency (or) Transit Time Noise :-

In semiconductor devices, the transit time is the time taken by the carrier to cross a junction. The periodic time of the signal is equal to reciprocal of signal frequency.

NOISE FACTOR AND NOISE FIGURE :-

NOISE FACTOR (F) :-

Noise Factor (F) of an amplifier, or any other network is defined as ratio of signal to noise power ratio at the input to the signal to noise power ratio at the output is called noise factor.

$$\text{Noise Factor } (F) = \frac{(SNR)_i}{(SNR)_o} \quad \text{—} (1)$$

W.K.T

$(SNR)_i$:- Signal to Noise Ratio at the input :-

It is defined as the ratio of signal power at input (P_{si}) to the Noise power at the input (P_{ni}).

$$\therefore (SNR)_i = \frac{\text{Signal power at input}}{\text{Noise power at input}}$$

$$\boxed{(SNR)_i = \frac{P_{si}}{P_{ni}}}$$

$(SNR)_o$:- Signal to Noise Ratio at the Output :-

It is defined as the ratio of signal power at output (P_{so}) to the Noise power at output (P_{no}).

$$\therefore \boxed{(SNR)_o = \frac{P_{so}}{P_{no}}}$$

$\therefore$ Eq. (1) becomes,

$$\boxed{F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}} \quad \text{— (2)}$$

In many cases, the noise factor 'F' is always unity depends on frequency and it is calculated at one signal frequency it is known as Spot noise factor.

Power Gain 'G' of the circuit is,

$$G = \frac{\text{Signal power at output}}{\text{Signal power at input}} = \frac{P_{so}}{P_{si}}$$

$\therefore$ eq. (2) becomes,

$$F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}} \qquad \left[\because G = \frac{P_{so}}{P_{si}}\right]$$

$$F = \frac{\cancel{P_{si}}}{\cancel{P_{ni}}} \times \frac{1}{G}$$

$$F = \frac{P_{no}}{P_{ni}} \times \frac{1}{G}$$

$P_{no} = P_{ni} \times F \times G$

$$\boxed{P_{no} = FG\,P_{ni}} \quad -③$$

W.K.T; Noise power at input (Thermal noise)

$$\boxed{P_{ni} = KTB}$$

∴ eq ③ becomes,

$$\boxed{P_{no} = FGKTB} \quad -④$$

Where,

- $F \rightarrow$ Noise Factor
- $G \rightarrow$ Gain in the amplifier
- $K \rightarrow$ Boltzmann's constant
- $T \rightarrow$ Temperature in °K
- $B \rightarrow$ Bandwidth in Hz

NOISE FIGURE:-

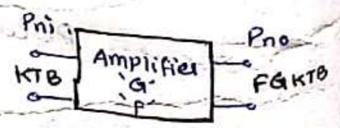
Noise Factor is expressed in decibel, it is known as "noise figure".

$$\boxed{\text{Noise Figure} = 10 \log [F]}$$

W.K.T, from eq. ④

$P_{no} = FGKTB$

∴ Total noise power at input



$P_{ni} = KTB$

From ③, $P_{ni} = \dfrac{P_{no}}{G} = \dfrac{FKTB\cancel{G}}{\cancel{G}}$

$$\boxed{P_{ni} = FKTB}$$

Source contributes 'KTB' and hence the amplifier contributes noise power, P_{na} gives by,

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P_{na} = Total P_{ni} − P_n due to source.

$P_{na} = FKTB - KTB$

$$\boxed{P_{na} = (F-1)KTB}$$

The fraction of total available noise contributed by the amplifier,

$$= \frac{(F-1)\cancel{KTB}}{F\cancel{KTB}} = \frac{(F-1)}{F}$$

NOISE TEMPERATURE :-

DEFINITION:-

The available noise power is directly proportional to temperature and it is independent of value of resistance. This power specified in terms of temperature is called noise temperature.

W.K.T

Noise power due to amplifier, having a noise factor F is

$$\boxed{P_{na} = (F-1)KTB} \quad \text{—} (1)$$

If $T_e \rightarrow$ Equivalent noise temperature represent noise power,

From Definition,

$$\boxed{P_{na} = KT_e B} \quad \text{—} (2)$$

To determine Noise temperature T_e, equating (1) = (2)

$$\cancel{K}T_e \cancel{B} = (F-1)\cancel{K}T\cancel{B}$$

$$\boxed{T_e = (F-1)T} \quad \text{—} (3)$$

Using Friss formula,

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots \quad — (4)$$

Subtract (1) on both side in eq. no. (4)

$$F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots \quad — (5)$$

From the noise Temperature,

$$T_e = (F-1) T$$

$$(F-1) = \frac{T_e}{T}$$

sub $(F-1) = T_e/T$ in eq. no. (5)

$$\therefore \frac{T_e}{T} = \frac{T_{e1}}{T} + \frac{\left(\frac{T_{e2}}{T}\right)}{G_1} + \frac{\left(\frac{T_{e3}}{T}\right)}{G_1 G_2} + \cdots$$

$$\boxed{T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots}$$

NOISE IN CASCADE SYSTEM:- [OR] "FRISS'S FORMULA"

Consider two amplifier connected in cascade. For one amplifier, the power gain is G_1 and its noise factor F_1; while the corresponding figures for the other amplifier are G_2 and F_2, respectively. The figure shown below,

When the amplifier consists of more than one stage its noise figure can be computed in terms of noise figure of individuals stages

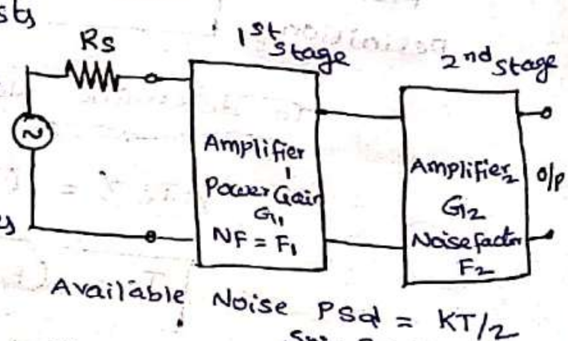


Fig: Cascade Connection of Amplifier

First determine (S_{no}) ie, Available total noise Power at output terminals,

S_{no} consists of two Components,

$S_1 \rightarrow$ Total noise power density available at the output due to 1^{st} stage noise.

$S_2 \rightarrow$ Total noise power density available at the output due to 2^{nd} stage noise.

Noise Psd (Power Spectral density) from any two terminal network can be written as,

$$\boxed{S_{ni} = \frac{KT}{2} = P_{ni}} \quad — ①$$

W.K.T, Noise Factor (F), from eq. ③ in noise factor

$$\boxed{F = \frac{P_{no}}{G P_{ni}}} \quad — ②$$

sub ① in ② ⇒ becomes,

$$F = \frac{P_{no}}{G\,(KT/2)} = \frac{2P_{no}}{GKT}$$

$$\therefore \boxed{P_{no} = \frac{FGKT}{2}}$$

Then psd at output due to 1^{st} stage of amplifier is written as,

$$\boxed{S_1 = \frac{F_1 G_1 KT}{2}}$$

Let the noise Figure of amplifier 2 is F_2. The noise Psd at the output due to 2^{nd} stage of amplifier S_2 is given as,

$$\therefore \boxed{S_2 = \frac{F_2 G_2 KT}{2} - \frac{G_2 KT}{2}}$$

1st term $\rightarrow \frac{F_2 G_2 KT}{2} \rightarrow$ Total noise Power density at output of 2nd stage.

2nd term $\rightarrow \frac{G_2 KT}{2} \rightarrow$ Noise Power density due to Second amplifier.

$$\therefore \boxed{S_2 = \frac{G_2 KT}{2}(F_2 - 1)}$$

and $S_{21} \rightarrow$ noise power due to amplifier 1 which is amplified by amplifier '2'.

$$\boxed{S_{21} = \frac{F_1 KT G_1 G_2}{2}}$$

Hence, Total noise power at output

$$(S_{no})_{av} = S_{21} + S_2$$

$$(S_{no})_{av} = \frac{G_2 KT}{2}(F_2 - 1) + \frac{F_1 KT G_1 G_2}{2}$$

$$\boxed{(S_{no})_{av} = \frac{KT}{2}\left[(F_2 - 1) G_2 + F_1 G_1 G_2\right]}$$

Where,

$$G_{ab} = G_1 G_2$$

$$F_{ab} = F_1 + \frac{F_2 - 1}{G_1}$$

$$\boxed{(S_{No})_{av} = \frac{KT}{2} G_1 G_2 \left[F_1 + \frac{(F_2 - 1)}{G_1}\right]} \quad \text{--- (3)}$$

For, multistage amplifier, the noise factor is given as,

$$F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \cdots$$

From above equation, noise Factor 'F' is depends on gain of the amplifier stage. It is known as "Friss's Formula".

NARROW BAND NOISE - PSD OF INPHASE AND QUADRATURE NOISE :-

Definition:-

The receiver of a communication system usually includes some provision for preprocessing the received signal. The preprocessing ~~the~~ may take the form of a narrow band filter whose bandwidth is just large enough to pass the modulated component of a received signal.

The noise appearing at the output of such a filter is called narrow band noise. The spectral component of narrow band noise concentrated about some mid-band frequency $\pm f_c$.

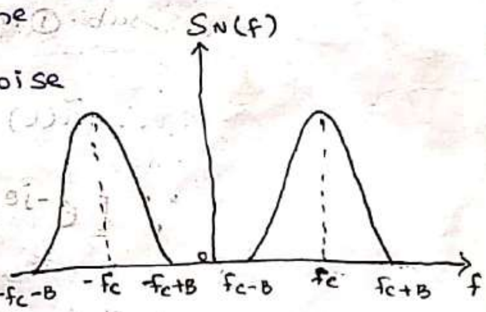


Fig:- PSD of Narrow Band Noise.

To analyze the effect of narrow band noise on the performance of a communication system, we need a mathematical representation of it. Depending on the application of interest, there are two specific representation of NBN.

(1) In terms of a pair of component called inphase and quadrature component.

(2) Represented in terms of Envelope and Phase.

The sample function of such narrow band noise $n(t)$ appears as a sine wave of frequency f_c which modulates slowly in amplitude and phase.

$$\boxed{S_N(f) = \frac{N_0}{2}|H(f)|^2}$$

Representation of Narrow Band noise in terms of inphase and Quadrature Components:-

Generally, $n(t)$ can be represented by its pre-envelope and Complex envelope as follows,

Pre envelope and Complex envelope in terms of signal $\Bigg\} \Rightarrow$

$$x_p(t) = x(t) + j\hat{x}(t)$$

$$x_c(t) = x_p(t) \cdot e^{-j2\pi f_c t}$$

Similarly in terms of noise, pre-envelope and Complex envelope is represented as,

Pre envelope $n_+(t) = n(t) + j\hat{n}(t)$ —①

Complex $\tilde{n}(t) = n_+(t) \cdot e^{-j2\pi f_c t}$ —②

sub. ① in ②

$$\therefore \tilde{n}(t) = [n(t) + j\hat{n}(t)] \cdot e^{-j2\pi f_c t}$$

$$\because [e^{-j\theta} = \cos\theta - j\sin\theta]$$

$$\therefore \tilde{n}(t) = [n(t) + j\hat{n}(t)][\cos 2\pi f_c t - j\sin 2\pi f_c t]$$

$$= n(t)\cos 2\pi f_c t - j\,n(t)\sin 2\pi f_c t + j\hat{n}(t)\cos 2\pi f_c t + \hat{n}(t)\sin 2\pi f_c t$$

$$\boxed{\tilde{n}(t) = n_c(t) + jn_s(t)}$$

where,

real Inphase Component of NBN $\Big\} \rightarrow n_c(t) = n(t)\cos 2\pi f_c t + \hat{n}(t)\sin 2\pi f_c t$

2 imaginary Quadrature Component of NBN $\Big\} \rightarrow n_s(t) = \hat{n}(t)\cos 2\pi f_c t - n(t)\sin 2\pi f_c t$

Eliminating $\hat{n}(t)$, hence,

$$\boxed{n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t} \quad \text{--- (1)}$$

The Psd of $n_c(t)$ and $n_s(t)$ are same and given as,

$$S_{nc}(f) = S_{ns}(f) = S(f_c - f) + S(f_c + f)$$

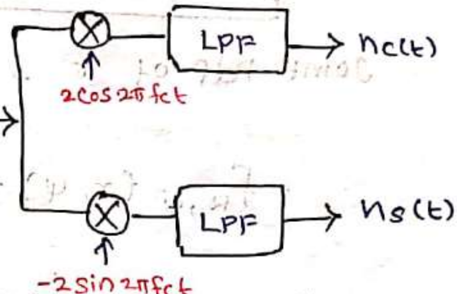
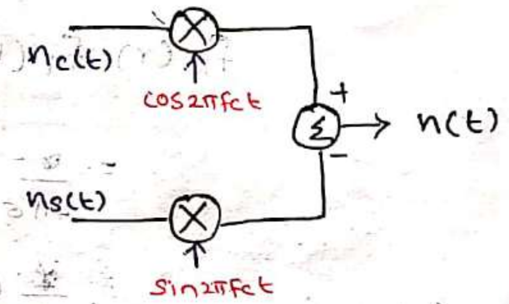


Fig: Extraction of inphase & quadrature

PROPERTIES OF NARROW BAND NOISE:-

(1) The in phase component $n_c(t)$ and quadrature component $n_s(t)$ of narrow band noise $n(t)$ have zero mean.

(2) If $n(t)$ is gaussian, then its $n_c(t)$ and $n_s(t)$ are jointly gaussian in nature.



(3) If $n(t)$ is WSS (wide sense stationary), then $n_c(t)$ and $n_s(t)$ are jointly stationary.

Fig: Generation of narrow band noise from inphase & quadrature component

(4) The PSD of $n_c(t)$ and $n_s(t)$ are same as that of $n(t)$.

(5) The cross spectral density of quadrature component of narrow band noise are purely imaginary.

$$\boxed{S_{ncns}(f) = -S_{ncns}(f)}$$

Representation of Narrow Band noise in terms of Envelope and Phase:-

$n_c(t) \rightarrow$ Amplitude of cosine component (in phase)

$n_s(t) \rightarrow$ Sine component (quadrature)

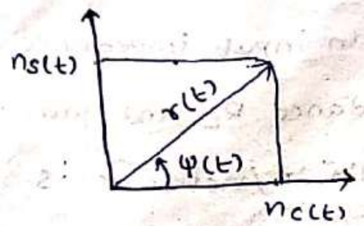


Fig: Phasor diagram of quadrature rep. of noise.

The resultant of two phasor is,

$$\boxed{\begin{aligned} r(t) &= \sqrt{n_c^2(t) + n_s^2(t)} \\ \psi(t) &= \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right) \end{aligned}}$$

Joint pdf of r & ψ is,

$$f_{R,\psi}(r,\psi) = \frac{r}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

$F_R(r) \cdot F_\psi(\psi$

R & ψ are statistically independent, joint pdf $F_{R,\psi}(r,\psi)$ is product of individuals.

$$f_R(r) \cdot f_\psi(\psi) = f_{R,\psi}(r,\psi)$$

$$= \frac{r}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

$$= \frac{r}{\sigma^2} e^{-r^2/\sigma^2} \cdot \frac{1}{2\pi}$$

Here,

$$f_\psi(\psi) = \begin{cases} \frac{1}{2\pi} & ; 0 \leq \psi \leq 2\pi \\ 0 & ; \text{otherwise} \end{cases}$$ ← Phase Component (ψ) Uniformly distributed

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} & ; r \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$ Envelope component (r) Rayleigh distribution

EXAMPLE FOR NOISE FACTOR:-

Let us consider a network, has an input impedance R_i; output impedance R_L and gain of the amplifier / network is 'A'.

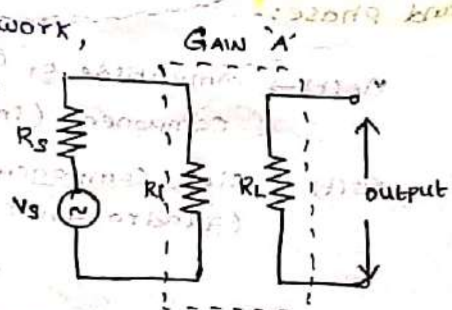


Fig: Noise Figure Measurement

To determine noise figure of the circuit, following steps

STEPS :-

(1) Determination of Input signal Power (P_{si})

(2) Determination of Input noise power (P_{ni})

(3) Calculate Input $(SNR)_i = \frac{P_{si}}{P_{ni}}$

(4) Determination of output signal Power (P_{so})

(5) Determination of output noise Power (P_{no})

(6) Calculate Output $(SNR)_o = \frac{P_{so}}{P_{no}}$

(7) Noise Factor $(F) = \frac{(SNR)_i}{(SNR)_o}$

Step 1 :- Input signal power (P_{si}) :-

By using voltage divider rule,

$$V_{si} = \frac{V_s \cdot R_i}{R_s + R_i}$$

$$P_{si} = \frac{V_{si}^2}{R_i} = \frac{V_s^2 R_i^{\cancel{2}}}{\cancel{R_i}(R_s+R_i)^2}$$

$$\boxed{P_{si} = \frac{V_s^2 R_i}{(R_s+R_i)^2}}$$

where,
$R_i \rightarrow$ Input Resistance
$R_s \rightarrow$ Source Resistance.

Step 2: Determination of Input noise power (P_{ni}) :-

W.K.T $V_{in} = \sqrt{4KTBR}$

From Thermal noise,

From ckt. Here, $R = \frac{R_s \cdot R_i}{R_s + R_i}$

$$\therefore V_{ni} = \sqrt{4KTB \frac{R_s \cdot R_i}{R_s + R_i}}$$

$$P_{ni} = \frac{V_{ni}^2}{R_i} = \frac{4KTB\left(\frac{R_s \cdot \cancel{R_i}}{R_s + R_i}\right)}{\cancel{R_i}}$$

$$\boxed{P_{ni} = \frac{4KTB\,R_s}{R_s + R_i}}$$

S3) Calculate signal to noise ratio at Input $(SNR)_i$

$$\therefore (SNR)_i = \frac{P_{si}}{P_{ni}}$$

$$= \frac{V_s^2 \cdot R_i}{(R_s+R_i)^{\cancel{2}}} \cdot \frac{\cancel{(R_s+R_i)}}{4KTB R_s}$$

$$\boxed{(SNR)_i = \frac{V_s^2 \cdot R_i}{4KTB R_s (R_s+R_i)}}$$

S4) Determination of Output Signal Power (P_{so}) :-

$$P_{so} = \frac{V_{so}^2}{R_L} = \frac{A \cdot V_{si}^2}{R_L}$$ At matched condition

$$\therefore [V_{so}^2 = V_{si}^2]$$

$$\therefore P_{so} = \frac{A\left(\frac{V_s^2 R_i^2}{(R_s+R_i)^2}\right)}{R_L}$$

S5) Output noise power (P_{no}) :-

Output noise Power is denoted as P_{no}.

S6) Calculate signal to noise ratio at Output $(SNR)_o$

$$\therefore (SNR)_o = \frac{P_{so}}{P_{no}}$$

$$\boxed{(SNR)_o = \frac{A \cdot V_s^2 \cdot R_i^2}{(R_s+R_i)^2 \cdot R_L \cdot P_{no}}}$$

S7) Noise Factor (F)

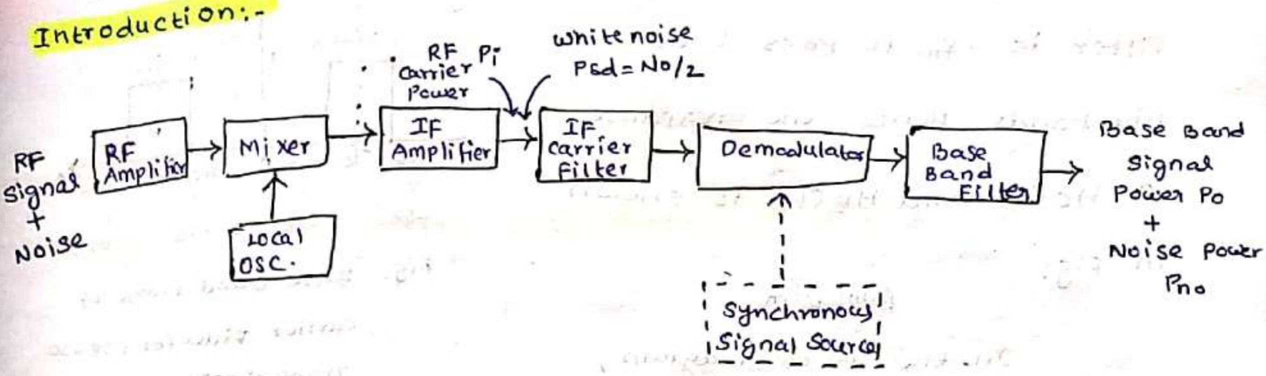
$$F = \frac{(SNR)_i}{(SNR)_o}$$

$$= \frac{\cancel{V_s^2} \cdot \cancel{R_i}}{4KTB R_s \cancel{(R_s+R_i)}} \times \frac{(R_s+R_i)^{\cancel{2}} R_L \cdot P_{no}}{A \cdot \cancel{V_s^2} \cdot R_i^{\cancel{2}}}$$

$$\boxed{F = \frac{R_L \cdot P_{no} (R_s+R_i)}{4KTBA R_s R_i}}$$

NOISE PERFORMANCE IN AM SYSTEM:-

Introduction:-



Generalized block diagram of AM signal receiver which uses Superheterodyne principle. The mixer converts the incoming RF signal in to IF (Intermediate Frequency). Then it is passed to IF amplifier. IF signal has a power P_i. If the noise of IF is white, then psd of this white noise will be $S_n(f) = N_0/2$. Then demodulator recovers the base band signal (ie, modulating signal).

I

NOISE PERFORMANCE IN DSB-SC-AM RECEIVER (or) COHERENT DETECTION (or) Synchronous demodulator:-

In DSB-SC-AM, the range of transmitted frequency will be $f_c - f_m$ to $f_c + f_m$. Here $f_c - f_m$ to f_c represent LSB and f_c to $f_c + f_m$ represent USB. For DSB-SC shown as,

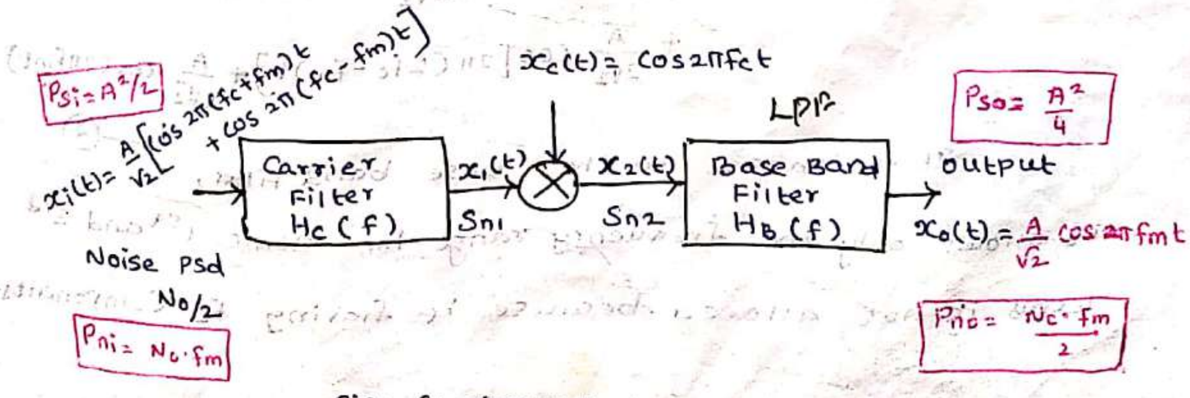


Fig: Synchronous Demodulator For DSB-SC

The Band width of the carrier Filter is $2f_m$ to Pass both the Sidebands. Hence the response of $H_C(f)$ and $H_B(f)$ is shown in Fig:

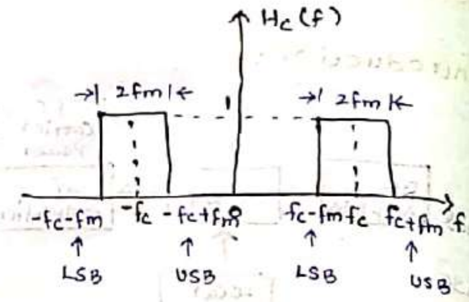


Fig: Base Band range of Carrier Filter for DSB-SC Transmission.

$B.w = 2f_m$

In the block diagram, the input modulated signal is represented as,

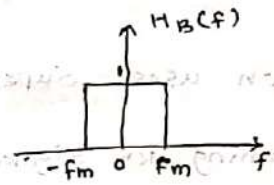


Fig: Base Band Filter, it Passes the frequency < fm.

$$\boxed{x_i(t) = \frac{A}{\sqrt{2}}\cos[2\pi(f_c+f_m)t] + \frac{A}{\sqrt{2}}\cos[2\pi(f_c-f_m)t]} \quad (1)$$

In above eq: Two cosine terms represents two side bands f_c+f_m and f_c-f_m. From the block $x_i(t) = x_1(t)$.

The output of multiplier,

$$x_2(t) = x_1(t) \cdot x_c(t). \quad \text{[From block diagram]}$$

$$x_2(t) = \frac{A}{\sqrt{2}}\cos 2\pi(f_c+f_m)t \cdot \cos 2\pi f_c t + \frac{A}{\sqrt{2}}\cos 2\pi(f_c-f_m)t \cos 2\pi f_c t$$

$$\left[\because 2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)\right]$$

$$x_2(t) = \frac{A}{2\sqrt{2}}\cos[2\pi(2f_c+f_m)t] + \frac{A}{2\sqrt{2}}\cos 2\pi f_m t$$
$$+ \frac{A}{2\sqrt{2}}\cos[2\pi(2f_c-f_m)t] + \frac{A}{2\sqrt{2}}\cos(2\pi f_m t) \quad (2)$$

It is passed to the base band filter, it allows only the frequency range fm. Hence 1st and 3rd terms is not allowed because it having fc Components

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$$\therefore\ x_0(t) = \frac{A}{2\sqrt{2}} \cos 2\pi f_m t + \frac{A}{2\sqrt{2}} \cos 2\pi f_m t$$

$$\boxed{x_0(t) = \frac{A}{\sqrt{2}} \cos 2\pi f_m t}$$

To calculate the Performance measures of DSB-SC-AM:-

STEP 1: Input signal Power (P_{si})

STEP 2: Input Noise Power (P_{ni})

STEP 3: Signal to Noise Ratio at input $(SNR)_i = \frac{P_{si}}{P_{ni}}$

STEP 4: Output signal Power (P_{so})

STEP 5: Output Noise Power (P_{no})

STEP 6: Signal to Noise ratio at output $(SNR)_o = \frac{P_{so}}{P_{no}}$

STEP 7: Figure of Merit $(\gamma) = \frac{(SNR)_o}{(SNR)_i}$

STEP 1: INPUT SIGNAL POWER (P_{si}):-

Generally, Power is expressed as,

$$\boxed{P_{si} = \frac{V_{rms}^2}{R}} \quad ; \quad \text{where } V_{rms} = \frac{\text{Amplitude of Signal}}{\sqrt{2}}$$

$R = 1$; For normalized signal,

From eq. (1), the signal Voltage (Amplitude) is $\frac{A}{\sqrt{2}}$ for both side band.

$$\therefore\ V_{rms}^2 = \left[\frac{(A/\sqrt{2})}{\sqrt{2}}\right]^2 = \frac{A^2}{4}$$

For DSB,

$$V_{rms}^2 = \frac{A^2}{4} + \frac{A^2}{4} = \frac{A^2}{2}$$

$$\therefore\ \boxed{P_{si} = \frac{A^2}{2}} \quad \text{—(3)}$$

STEP 2: INPUT NOISE POWER (P_{ni}):-

Consider, $S_{ni} = \frac{N_0}{2}$

From block,

$$\therefore P_{ni} = \int_{-f_m}^{f_m} S_{ni}(f) \cdot df$$

$$= \int_{-f_m}^{f_m} \frac{N_0}{2} \cdot df$$

$$= \frac{N_0}{2} [f]_{-f_m}^{f_m}$$

$$= \frac{N_0}{2} [f_m + f_m]$$

$$\therefore P_{ni} = \frac{N_0}{\cancel{2}} \cdot \cancel{2} f_m$$

$$\boxed{P_{ni} = N_0 \cdot f_m} \quad — (4)$$

STEP 3:- SIGNAL TO NOISE RATIO AT INPUT $(SNR)_i$

$$\therefore (SNR)_i = \frac{P_{si}}{P_{ni}} = \frac{\text{Input Signal Power}}{\text{Input Noise Power}}$$

sub. (3) & (4) ⇒

$$(SNR)_i = \frac{A^2}{2} \times \frac{1}{N_0 \cdot f_m}$$

$$\Rightarrow \boxed{(SNR)_i = \frac{A^2}{2N_0 \cdot f_m}} \quad — (5)$$

STEP 4:- OUTPUT SIGNAL POWER (P_{so}):-

Let we know the output signal of the demodulator, (ie)

$$x_0(t) = \frac{A}{\sqrt{2}} \cos 2\pi f_m t$$

$$\therefore P_{so} = \frac{V_{rms}^2}{R} = \frac{\left[\left(A/\sqrt{2}\right)/\sqrt{2}\right]^2}{1}$$

$$\boxed{P_{so} = \frac{A^2}{4}} \quad — (6)$$

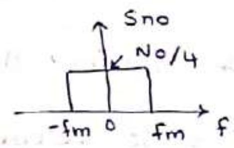
Ratio between output/Input signal Power P_{so}/P_{si}

$$\frac{A^2}{\cancel{4}_2} \times \frac{\cancel{2}}{A^2} = \frac{1}{2} \text{ ratio};$$ because it is DSB-SC.

STEP 5:- OUTPUT NOISE POWER :- (P_{no})

$$S_n(f_c+f_m) = S_n(f_c-f_m) = \frac{S_n(f)}{2}$$

W.K.T $S_n(f) = \frac{N_0}{2}$ (white noise)



$$\therefore S_n(f_c+f_m) = S_n(f_c-f_m) = \frac{N_0}{2} \times \frac{1}{2}$$

$$\boxed{S_n(f) = N_0/4}$$

$\therefore$ output Noise Power (P_{no})

$$P_{no} = \int_{-f_m}^{f_m} S_n(f) \cdot df$$

$$= \int_{-f_m}^{f_m} \frac{N_0}{4} \cdot df$$

$$= \frac{N_0}{\cancel{4}_2} [\cancel{2} f_m]$$

$$\boxed{P_{no} = \frac{N_0 \cdot f_m}{2}} \quad — (7)$$

STEP 6:- SIGNAL TO NOISE RATIO AT OUTPUT $(SNR)_o$

$$\therefore (SNR)_o = \frac{P_{so}}{P_{no}} = \frac{\text{Output signal Power}}{\text{Output Noise power}}$$

sub (6) & (7) $\Rightarrow$

$$(SNR)_o = \frac{A^2}{\cancel{4}_2} \times \frac{\cancel{2}}{N_0 \cdot f_m}$$

$$\Rightarrow \boxed{(SNR)_o = \frac{A^2}{2 N_0 \cdot f_m}} \quad — (8)$$

STEP 7:- FIGURE OF MERIT (γ)

$$FOM\ (\gamma) = \frac{(SNR)_o}{(SNR)_i}$$

sub. (5) & (8) $\Rightarrow$

$$FOM(\gamma) = \frac{\cancel{A^2}}{\cancel{2 N_0 f_m}} \times \frac{\cancel{2 N_0 f_m}}{\cancel{A^2}}$$

$$\boxed{\gamma = 1}$$

Since, Figure of merit of DSB-SC is unity (1), No improvement in SNR.

NOISE PERFORMANCE IN SSB-SC-AM RECEIVER (or) COHERENT RECEIVER (or) Synchronous Receiver:-

In SSB-SC-AM, the range of transmitted frequency will be $f_c + f_m$. Here f_c to $f_c + f_m$ represent USB only. The block of SSB-SC receiver as shown.

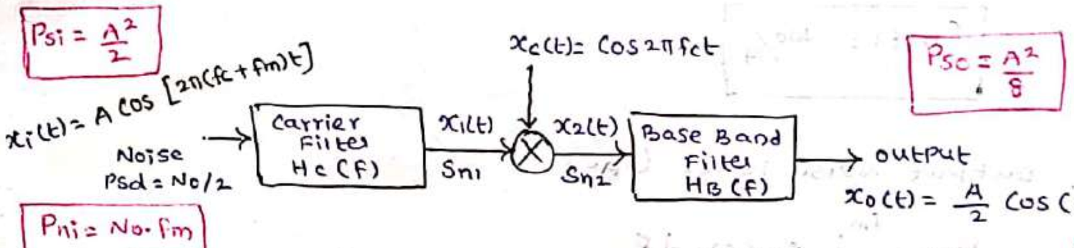


Fig: Synchronous Demodulator for SSB-SC-AM

$P_{nc} = \frac{N_0 f_m}{4}$

The frequencies from f_c to $f_c + f_m \rightarrow$ USB.

Lower side band is suppressed. The modulating signal is sinusoidal having frequency f_m. Then the modulated signal $x_i(t)$ with SSB-SC-AM can be written as,

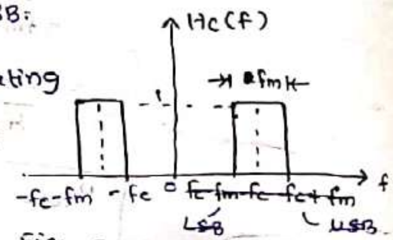


Fig: Base band range of Carrier Filter. It passes frequency f_c to $f_c + f_m$ (USB)

$$\boxed{x_i(t) = A\cos[2\pi(f_c+f_m)t]} \quad \text{--- (1)}$$

In above eq. the cosine term represent USB, (ie) $x_i(t) = x_1(t)$.

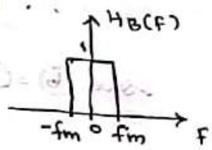


Fig: Low pass baseband Filter.

The output of multiplier is,

$$x_2(t) = x_1(t) \cdot x_c(t) \quad \text{[From block diagram]}$$

$$x_2(t) = A\cos[2\pi(f_c+f_m)t]\cos 2\pi f_c t$$

$$x_2(t) = \frac{A}{2}\cos[2\pi(2f_c+f_m)t] \qquad \left[\because \cos A\cos B = \frac{\cos(A+B)+\cos(A-B)}{2}\right]$$

$$+ \frac{A}{2}\cos[2\pi f_m t]$$

It is passed to the base band filter, it allows only the frequency range f_m. Hence from the above eq. 1st term is eliminated because it having f_c components.

$$\therefore \boxed{x_0(t) = \frac{A}{2} \cos 2\pi f_m t} \quad \text{—(2)}$$

The detected signal $x_0(t)$ contains only modulating frequency f_m. At the same time, amplitude of modulated signal is reduce to half (ie) $A/2$, because it is SSB-SC transmission.

To calculate the Performance Measures of SSB-SC-AM:-

STEP 1:- Input signal Power (P_{si})

STEP 2:- Input noise Power (P_{ni})

STEP 3:- Signal to Noise ratio at input $(SNR)_i = \frac{P_{si}}{P_{ni}}$

STEP 4:- Output Signal Power (P_{so})

STEP 5:- Output Noise Power (P_{no})

STEP 6:- Signal to Noise ratio at Output $(SNR)_o = \frac{P_{so}}{P_{no}}$

STEP 7:- Figure of merit $(\gamma) = \frac{(SNR)_o}{(SNR)_i}$

STEP 1: Input signal power (P_{si}):-

Generally, Power is expressed as,

$$\boxed{P_{si} = \frac{V_{rms}^2}{R}} \quad \text{where } V_{rms} = \frac{\text{Amplitude}}{\sqrt{2}}$$

$R = 1$, for normalized signal

From eq. (1), the signal Voltage (Amplitude) is 'A'.

$$\therefore P_{si} = \frac{(A/\sqrt{2})^2}{1}$$

$$\boxed{P_{si} = \frac{A^2}{2}} \quad \text{—(3)}$$

STEP 2: Input Noise Power:-

Consider, $S_{ni} = N_0/2$

From block,

$$\therefore P_{ni} = \int_{-f_m}^{f_m} S_{ni}(f) \cdot df$$

$$= \int_{-f_m}^{f_m} N_0/2 \cdot df = \frac{N_0}{\cancel{2}} (\cancel{2} f_m)$$

$$\boxed{P_{ni} = N_0 \cdot f_m} \quad \text{—(4)}$$

STEP 3: Signal to Noise Ratio at input $(SNR)_i$:-

$$(SNR)_i = \frac{P_{si}}{P_{ni}} = \frac{\text{Input signal power}}{\text{Input noise power}}$$

sub ③ & ④ ⇒

$$(SNR)_i = \frac{A^2}{2} \times \frac{1}{N_0 \cdot f_m}$$

$$\Rightarrow \boxed{(SNR)_i = \frac{A^2}{2N_0 \cdot f_m}} \quad — ⑤$$

STEP 4:- Output signal Power:- (P_{so})

Let we know the output signal of the demodulator is, (ie)

$$x_0(t) = \frac{A}{2} \cos 2\pi f_m t$$

$$\therefore P_{so} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{A/2}{\sqrt{2}}\right)^2}{1}$$

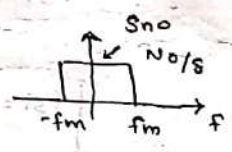
$$\boxed{P_{so} = \frac{A^2}{8}} \quad — ⑥$$

Ratio between $\frac{\text{output}}{\text{Input}}$ signal power $= \frac{P_{so}}{P_{si}} = \frac{A^2}{8_4} \times \frac{2}{A^2} = 1/4$ ratio,

because it is SSB-SC.

STEP 5:- Output Noise Power (P_{no}):-

The noise $n(t)$ having Power spectral density (psd) $S_n(f)$ is multiplied with $\cos 2\pi f_c t$; then resultant signal becomes $n(t) \cdot \cos 2\pi f_c t$.



$$S_n(f_c + f) = S_n(f_c - f) = \frac{S_n(f)}{4}$$

$$= \frac{N_0}{2} \times \frac{1}{4}$$

$$\boxed{S_n(f_c+f) \text{ (or) } S_n(f) = \frac{N_0}{8}}$$

$$\therefore P_{no} = \int_{-f_m}^{f_m} S_n(f) \cdot df = \int_{-f_m}^{f_m} \frac{N_0}{8} \cdot df$$

$$P_{no} = \frac{N_0}{\cancel{8}_4} (\cancel{2} f_m)$$

$$\boxed{P_{no} = \frac{N_0 \cdot f_m}{4}} \quad \text{--- (7)}$$

STEP 6:- Signal to Noise Ratio at output $(SNR)_0$

$$(SNR)_0 = \frac{P_{so}}{P_{no}} = \frac{\text{Output signal power}}{\text{Output Noise Power}}$$

sub (6) & (7) ⇒

$$(SNR)_0 = \frac{A^2}{\cancel{8}_2} \times \frac{\cancel{4}}{N_0 \cdot f_m}$$

$$\Rightarrow \boxed{(SNR)_0 = \frac{A^2}{2 N_0 \cdot f_m}} \quad \text{--- (8)}$$

STEP 7:- FIGURE OF MERIT (γ):-

$$(FOM)\ \gamma = \frac{(SNR)_0}{(SNR)_i}$$

sub (5) & (8) ⇒

$$\gamma = \frac{\cancel{A^2}}{\cancel{2N_0 f_m}} \times \frac{\cancel{2N_0 f_m}}{\cancel{A^2}}$$

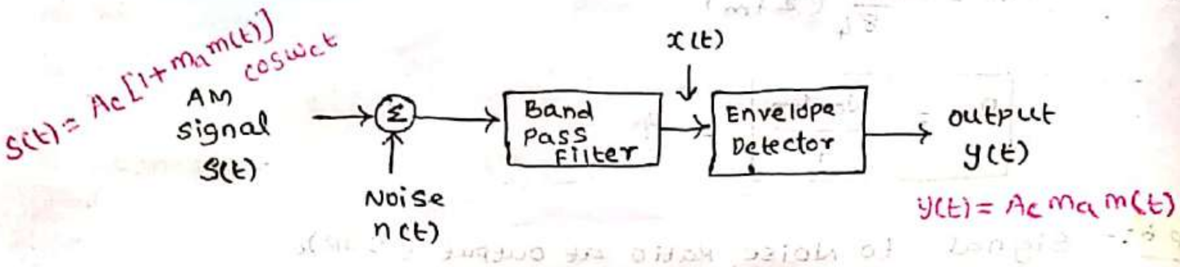
$$\boxed{\gamma = 1}$$

∴ Since, Figure of merit of SSB-SC-AM is unity 1.

Hence there is no improvement in signal to Noise ratio.

iii) NOISE PERFORMANCE IN AM RECEIVER USING ENVELOPE DETECTOR (or) NON-COHERENT RECEIVER:-

Fig: AM RECEIVER MODEL USING ENVELOPE DETECTOR,



Consider the AM transmission that has both the sidebands and a full carrier. Such modulated signal is mathematically represents as,

$$\boxed{s(t) = A_c[1+m_a m(t)]\cos 2\pi f_c t} \quad \text{—(1)}$$

Where,

$A_c \cos 2\pi f_c t \rightarrow$ Carrier signal

$m(t) \rightarrow$ message signal

m_a (modulation index of AM).

The envelope detector consists of modulated message signal $s(t)$ plus noise $n(t)$.

(ie) From Fig:-

$$\boxed{x(t) = s(t) + n(t)} \quad \text{—(2)}$$

Here, $n(t) \rightarrow$ noise represent in terms of inphase and Quadrature Component.

$$\therefore n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \quad \text{—(3)}$$

sub (1) & (3) in eq. (2) $\Rightarrow$

$$\therefore x(t) = A_c[1+m_a m(t)]\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$$

$$x(t) = [A_c + A_c m_a m(t) + n_c(t)]\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$$

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For the above equation, phasor representation of $x(t)$ is

The resultant is the envelope of $x(t)$,

(ie) output of envelope detector. Hence,

$$y(t) = \sqrt{(A_c + A_c m_a m(t) + n_c(t))^2 + n_s^2(t)}$$

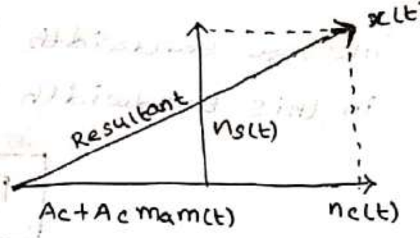


Fig: Phasor diagram for AM & noise

When signal power is large compared to noise power, then $n_s(t)$ and $n_c(t)$ will be very small compared to $A_c(1+m_a m(t))$.

$$\therefore \boxed{y(t) = A_c + A_c m_a m(t)} \quad \text{--- (4)}$$

$A_c \rightarrow$ removed by blocking capacitor after envelope detector

$$\therefore \boxed{y(t) = A_c m_a m(t)} \quad \text{--- (5)}$$

To calculate the performance measures of AM Receiver using Envelope detector:-

STEP 1: Input signal Power (P_{si})

STEP 2:- Input noise Power (P_{ni})

STEP 3:- Signal to Noise ratio at input $(SNR)_i = \frac{P_{si}}{P_{ni}}$

STEP 4:- Output Signal Power (P_{so})

STEP 5:- Output Noise Power (P_{no})

STEP 6:- Signal to Noise Ratio at Output $(SNR)_o = \frac{P_{so}}{P_{no}}$

STEP 7:- Figure of Merit $(\gamma) = \frac{(SNR)_o}{(SNR)_i}$

STEP 1: Input signal power (P_{si})

For AM, the modulated signal power P_{total} is,

$$P_{total} = P_c\left(1 + \frac{m_a^2}{2}\right)$$

where,

$P_c \rightarrow$ Carrier Power $= \frac{A_c^2}{2}$

$m_a \rightarrow$ modulation index

'p' $\rightarrow$ Average Power of message signal.

$$\boxed{P_{si} = \frac{A_c^2}{2}[1 + m_a^2 \cdot p]} \quad \text{--- (5)}$$

STEP 2: Input noise power (P_{ni})

Earlier we have proved that if the message bandwidth is 'B', then the average noise power in this bandwidth will be,

$$\boxed{P_{ni} = N_0 \cdot B} \quad — (6)$$

STEP 3: Signal to Noise Ratio at input $(SNR)_i$

$$(SNR)_i = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_{si}}{P_{ni}}$$

$$(SNR)_i = \frac{A_c^2}{2}(1 + m_a^2 P) \cdot \frac{1}{N_0 \cdot B}$$

$$\Rightarrow \boxed{(SNR)_i = \frac{A_c^2(1+m_a^2 P)}{2 N_0 \cdot B}} \quad — (7)$$

STEP 4: Output signal power (P_{so})

W.K.T output of envelope detector is

$$y(t) = A_c m_a m(t)$$

Then,

$$P_{so} = \frac{V_{rms}^2}{R} = \left(\frac{A_c m_a m(t)}{\sqrt{2}}\right)^2 / 1$$

$$\boxed{P_{so} = \frac{A_c^2 m_a^2 P}{2}} \quad — (8)$$

$P \rightarrow$ Avg. Power of message signal $m(t)$

STEP 5: Output noise power:- (P_{no}):-

Noise Power over the bandwidth 'B' is $N_0 \cdot B$.

$$\therefore \boxed{P_{no} = N_0 \cdot B} \quad — (9)$$

STEP 6: Signal to Noise ratio at output $(SNR)_o$

$$(SNR)_o = \frac{\text{output signal Power}}{\text{output noise power}} = \frac{P_{so}}{P_{no}}$$

$$(SNR)_o = \frac{A_c^2 m_a^2 P}{2} \times \frac{1}{N_0 \cdot B}$$

$$\Rightarrow \boxed{(SNR)_o = \frac{A_c^2 m_a^2 P}{2 N_0 \cdot B}} \quad \text{— (10)}$$

STEP7:- Figure of merit (γ)

$$\gamma = \frac{(SNR)_o}{(SNR)_i}$$

$$= \frac{\cancel{A_c^2} m_a^2 P}{\cancel{2N_0 \cdot B}} \times \frac{\cancel{2N_0 \cdot B}}{\cancel{A_c^2}(1+m_a^2 P)}$$

$$\boxed{\gamma = \frac{m_a^2 P}{1+m_a^2 P}} < 1$$

In envelope detector of AM receiver, the figure of merit 'γ' is always less than unity. Therefore, noise performance of DSB, SSB is better than AM receiver with envelope detector.

NOISE PERFORMANCE IN FM SYSTEM:-

To measure the performance of noise in FM receiver, the model is shown in Fig,

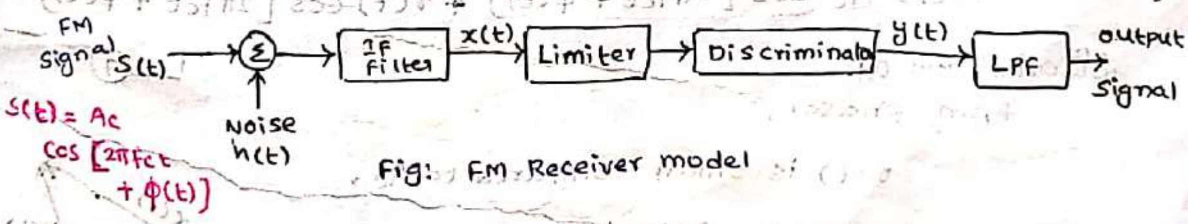


Fig: FM Receiver model

The narrow band noise $n(t)$ at the output of IF Filter can be represented in terms of in phase and Quadrature Components as,

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t \quad \text{— (1)}$$

The noise can be represented interms of envelope and phase (ie) $r(t)$ and $\varphi(t)$.

$$\boxed{n(t) = r(t)\cos[2\pi f_c t + \psi(t)]} \quad -②$$

Here,

W.K.T

$$\left.\begin{aligned} r(t) &= \sqrt{n_c^2(t) + n_s^2(t)} \\ \psi(t) &= \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right) \end{aligned}\right\} -③$$

The Frequency modulated signal is represented as,

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \phi(t)]} \quad -④$$

$n(t) = s(t)$
$r(t) = A_c$
$\psi(t) = \phi(t)$

Where,

$A_c \rightarrow$ Carrier amplitude

$\phi(t) \rightarrow$ phase deviation (Instantaneous)

The phase deviation $\phi(t)$ and message $m(t)$ are related as,

$$\phi(t) = 2\pi k_f \int_0 m(t) \cdot dt \quad -⑤$$

$k_f \rightarrow$ Frequency sensitivity.

$\therefore$ The output of IF Filter is,

$$x(t) = s(t) + n(t). \quad -⑥$$

sub. ④ & ② in ⑥ eq.

$$x(t) = A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \psi(t)] \quad -⑦$$

Discriminator o/p:

From phasor,

$r(t)$ is small compared to A_c,

then relative phase $\theta(t)$ is,

$$\theta(t) - \phi(t) = \frac{r(t)}{A_c}\sin(\psi(t) - \phi(t))$$

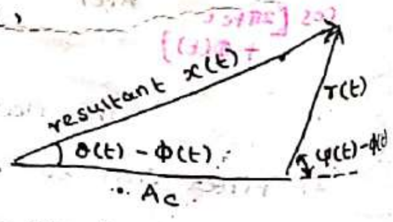


Fig: Phasor diagram.

$$① - \therefore \theta(t) = \phi(t) + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]$$

sub. $\phi(t)$ value in above equation, eqn ⑤

$$\theta(t) = 2\pi K_f \int_0^t m(t)\cdot dt + \frac{r(t)}{A_c} \sin(\psi(t) - \phi(t)) \quad — (8)$$

The output of discriminator $y(t)$, It is equal to the derivative of relative phase $\theta(t)$ divided by 2π. (ie)

$$\boxed{y(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t)}$$

$$y(t) = \frac{1}{2\pi} \left\{ 2\pi K_f\, m(t) + \frac{d}{dt} \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)] \right\}$$

$$y(t) = K_f\, m(t) + \frac{1}{2\pi A_c} \frac{d}{dt} \left\{ r(t) \sin[\psi(t) - \phi(t)] \right\}$$

The output depends on message (ie) $K_f \cdot m(t)$ and noise team $n_d(t)$.

$$\boxed{y(t) = K_f\, m(t)} \quad — (9)$$

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \left\{ r(t) \sin[\psi(t) - \phi(t)] \right\}$$

W.K.T.

$\psi(t) \Rightarrow$ Uniformly distributed over interval 0 to 2π

$\phi(t) \rightarrow$ It is a function of message signal

$$\therefore n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} r(t) \cdot \sin \psi(t) \quad — (10)$$

From eq. (2),

$$\left.\begin{aligned} n_s(t) &= r(t) \sin \psi(t) \\ n_c(t) &= r(t) \cos \psi(t) \end{aligned}\right\} — (11)$$

$\therefore$ eq. 10, becomes,

$$\therefore \boxed{n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_s(t)} \quad — (12)$$

To measure the Performance noise of FM System :-

Step 1: Output noise power (P_{no}):-

W.K.T $$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_s(t)$$

Differentiation property of Fourier Transform (F.T)

$$\frac{d}{dt} x(t) \overset{FT}{\longleftrightarrow} j2\pi f \; X(f)$$

This means $\frac{d}{dt} x(t)$ can be obtained by passing $x(t)$ through a filter with transform function $j2\pi f$ (ie) differentiator.

|||'ly $n_d(t)$ can be obtained as,

$$H(f) = \frac{1}{2\pi A_c} j2\pi f$$

$$\boxed{H(f) = j\frac{f}{A_c}}$$

If $S_{Ns}(f)$ is PSD of $n_s(t)$ and $S_{Nd}(f)$ is PSD of $n_d(t)$, then.

$$S_{Nd}(f) = |H(f)|^2 \cdot S_{Ns}(f)$$

$$\boxed{S_{Nd}(f) = \frac{f^2}{A_c^2} \cdot N_0}$$

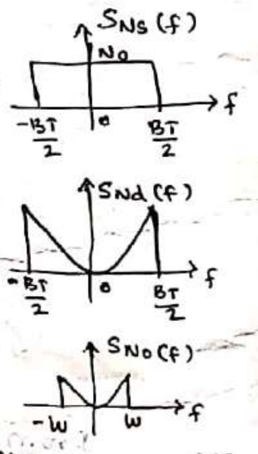


Fig: PSD of noise in FM Rx.

$$\therefore P_{no} = \int_{-W}^{W} S_{Nd}(f) \cdot df = \int_{-W}^{W} \frac{f^2}{A_c^2} \cdot N_0 \cdot df$$

$$= \frac{N_0}{A_c^2} \left[\frac{f^3}{3}\right]_{-W}^{W}$$

$$= \frac{N_0}{A_c^2} \left[\frac{W^3}{3} + \frac{W^3}{3}\right]$$

$$\boxed{P_{no} = \frac{2 N_0 W^3}{3 A_c^2}} \leftarrow (13)$$

STEP2: Input noise Power (P_{ni})

The average noise power in this bandwidth 'W' will be,

$$\boxed{P_{ni} = N_0 \cdot W} \quad \text{—(14)}$$

STEP3: Output Signal Power (P_{so}):

W.K.T from eqn (9)

output signal $y(t) = K_f\, m(t)$

$$\therefore P_{so} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{K_f m(t)}{\sqrt{2}}\right)^2}{1}$$

$$P_{so} = \frac{K_f^2\, m^2(t)}{2}$$

where

$$\therefore \boxed{P_{so} = K_f^2 \cdot P} \quad \text{—(15)}$$

$P \rightarrow$ Avg. of message signal (Power)

STEP4:- Input Signal Power (P_{si})

$$P_{si} = \frac{V_{rms}^2}{R} = \frac{(A_c/\sqrt{2})^2}{1}$$

$$\boxed{P_{si} = \frac{A_c^2}{2}} \quad \text{—(16)}$$

STEP5:- Signal to Noise Ratio at input (SNR)i

$$(SNR)_i = \frac{P_{si}}{P_{ni}} = \frac{A_c^2}{2} \cdot \frac{1}{N_0 \cdot W}$$

$$\Rightarrow \boxed{(SNR)_i = \frac{A_c^2}{2N_0 \cdot W}} \quad \text{—(17)}$$

STEP6:- Signal to Noise Ratio at Output (SNR)o

$$(SNR)_o = \frac{P_{so}}{P_{no}}$$

$$(SNR)_o = K_f^2 \cdot P \cdot \frac{3A_c^2}{2N_0 \cdot W^3}$$

$$\Rightarrow \boxed{(SNR)_o = \frac{3A_c^2 K_f^2 P}{2N_0 W^3}} \quad \text{—(18)}$$

Step 7:- Figure of merit $(\gamma) = \dfrac{(SNR)_o}{(SNR)_i}$

$$\gamma = \frac{(SNR)_o}{(SNR)_i} = \cancel{\frac{A_c^2}{2N_0 \cdot W}} \times$$

$$\gamma = \frac{3\cancel{A_c^2} K_f^2 \cdot P}{\cancel{2N_0} \cdot W^{\cancel{3}\,2}} \times \frac{\cancel{2N_0} \cdot \cancel{W}}{\cancel{A_c^2}}$$

$$\boxed{\gamma = \frac{3K_f^2 P}{W^2}}$$

W.K.T $\quad P = \dfrac{A_c^2}{2}$

$$\therefore \gamma = \frac{3K_f^2 \cdot \frac{A_c^2}{2}}{W^2} = \frac{3}{2}\frac{(K_f A_c)^2}{W^2} \qquad \left[\because \delta = K_f \cdot A_c\right]$$

$$\gamma = \frac{3}{2}\frac{\delta^2}{W^2} \qquad \left[\because \beta = \delta/W\right]$$

↳ modulation index

$$\boxed{\gamma = \frac{3}{2}\beta^2}$$

The deviation ratio is proportional to K_f/W. Hence the FoM is the quadratic function of deviation ratio.

The transmission Bandwidth B_T is proportional to deviation ratio. Hence increases in B_T, increases in FoM FM system.

PRE-EMPHASIS AND DE-EMPHASIS IN FM:-

Significance:-

The psd of message usually falls off at higher frequencies.

But psd of noise increases rapidly at higher frequency.

∴ Message signal is not

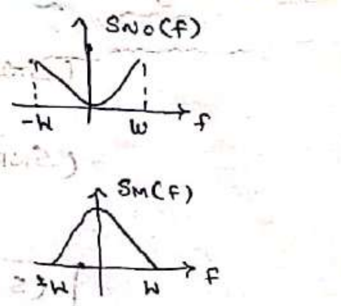


Fig: Psd of noise & message

utilizing the frequency band in efficient manner.

Such more efficient use of frequency band and improved noise performance can be obtained with help of Pre-emphasis and De-emphasis.

It also used to improve the threshold. The simple RC network are used to boost the high frequency at transmitter and attenuate them at the receiver.

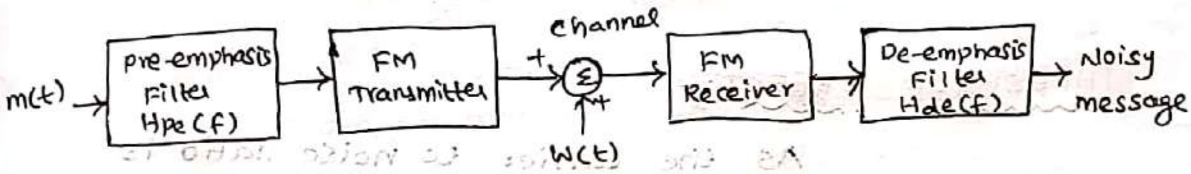


Fig: Pre-emphasis and De-emphasis in FM.

✓ The high frequency component are artificially emphasized by pre-emphasis filter before modulation.

✓ This equalizes the low frequency and high frequency portion of the psd and complete band is occupied.

✓ The FM signal is then transmitted. Noise w(t) is added to this signal before it reaches the receiver.

✓ De-emphasis restore the Power distribution of original signal. The high frequency component of noise are also reduced. This improves SNR (signal to noise ratio).

$$\boxed{H_{de}(f) = \frac{1}{H_{pe}(f)}} \quad -W \le f \le W.$$

CAPTURE EFFECT:-

The FM system minimizes the effect of noise interference. This can be effective when interference is weak compared to FM signal. But if the interference is stronger than FM signal, then FM receiver locks to interference. This suppresses FM signal.

When noise interference as well as FM signal are equal strength, then FM receiver locking fluctuates between them. This phenomenon is called capture effect.

THRESHOLD EFFECT:

As the carrier to noise ratio is reduced (CNR), clicks are heard in the receiver output. As (CNR) is reduced further, crackling, or sputtering sound appears at receiver output.

Near the breaking point the theoritical calculated output signal to noise ratio becomes large, but its actual value is very small. This phenomenon is called Threshold effect.

Problems:-

Thermal noise:-

1) Thermal noise from a resistor is measured as 4×10^{-17} W, for a given bandwidth and at a temperature of 20°C. What will be the noise power when temperature is changed to (i) 50°C (ii) 70°K

Given:-

$$P_n = 4 \times 10^{-17} \text{ W} \quad \text{at} \quad T = 20^\circ C$$

$$\therefore T = 20 + 273 = 293^\circ K$$

Sol

W.K.T

$$P_n = KTB \text{ watts} \qquad [K = 1.38 \times 10^{-23} \text{ J/K}]$$

$$4 \times 10^{-17} = 1.38 \times 10^{-23} \times 293 \times B$$

$$\boxed{B = 9892.66 \text{ Hz}}$$

(i) To obtain P_n; When $T = 50^\circ C$

$$P_n = KTB = 1.38 \times 10^{-23} \times (50+273) \times 9892.66$$

$$\boxed{P_n = 4.4 \times 10^{-17} \text{ W}}$$

(ii) To obtain P_n; $T = 70^\circ K$

$$P_n = KTB = 1.38 \times 10^{-23} \times 70 \times 9892.66$$

$$\boxed{P_n = 9.56 \times 10^{-18} \text{ Watts}}$$

2) Two resistors 20K and 50KΩ are at room temperature 27°C. Calculate for Bandwidth of 100KHz the thermal noise voltage (i) for each resistor (ii) for two resistor in series (iii) for two resistor in parallel.

Given:-

$$R_1 = 20K\Omega \;;\; R_2 = 50K\Omega \;;\; T = 27 + 273 = 300^\circ K$$

$$B = 100 \text{ KHz} \;;\; K = 1.38 \times 10^{-23} \text{ J/K}$$

constant

Sol

(i) Thermal noise voltage for each resistor

$$E_{n1} = \sqrt{4KTBR_1} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 100 \times 10^3 \times 20 \times 10^3}$$

$$\boxed{E_{n1} = 5.755\,\mu V}$$

$$E_{n2} = \sqrt{4KTBR_2} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 100 \times 10^3 \times 50 \times 10^3}$$

$$\boxed{E_{n2} = 9.1\,\mu V}$$

(ii) Series,

$$E_n = \sqrt{4KTB(R_1+R_2)} = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 100 \times 10^3 \times 70 \times 10^3}$$

$$\boxed{E_n = 10.77\,\mu V}$$

(iii) Parallel..

$$E_n = \sqrt{4KTB(R_1R_2/R_1+R_2)}$$

i.e. $R = \dfrac{R_1R_2}{R_1+R_2} = \dfrac{20 \times 50}{20+50} = 14.28\,k\Omega$

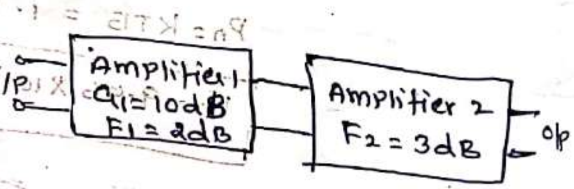
$$\boxed{E_n = 4.8\,\mu V}$$

Cascade:

1) Consider two amplifier are connected in cascade, 1st stage amplifier has gain and noise figure as 10dB and 2dB. Second stage has noise figure of 3dB. Calculate total noise figure.

Sol

Let us convert G_1, F_1, F_2 to linear scale,



$$G_1 dB = 10 \log_{10}(G_1)$$

$$10 = 10 \log G_1 \Rightarrow \boxed{G_1 = 10}$$

$$\therefore F_1 dB = 10 \log_{10}(F_1)$$

$$2 = 10 \log_{10} F_1 \Rightarrow \boxed{F_1 = 1.585}$$

$$\therefore F_2 dB = 10 \log_{10}(F_2)$$

$$3 = 10 \log_{10} F_2 \Rightarrow \boxed{F_2 = 2}$$

$\therefore$ Total noise figure (F)

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

$$F = 1.585 + \frac{(2-1)}{10}$$

$$\boxed{F = 1.685}$$

$$\therefore\ F = 10 \log_{10} (1.685)$$

$$\boxed{F = 2.266\ dB}$$

Sampling and Quantization

Lowpass Sampling - Aliasing - Signal Reconstruction - Quantization - Uniform & non-uniform quantization - quantization noise - Logarithmic Companding - PAM, PPM, PWM, PCM - TDM, FDM.

Sampling Theorem for low pass signals:

A continuous time signal can be completely represented in its samples and recovered back if the Sampling frequency is twice of the highest frequency of the signal

ie $f_s \geq 2f_m$.

Proof of Sampling Theorem:.

i) Representation of $x(t)$ in terms of its samples.
ii) Reconstruction of $x(t)$ from its samples.

i) Representation of $x(t)$ in its Samples.

Step 1: Define $x_\delta(t)$.
Step 2: Fourier transform of $x_\delta(t)$ ie $X_\delta(f)$.
Step 3: Relation between $X(f)$ and $X_\delta(f)$.
Step 4: Relation between $x(t)$ and $x(nT_s)$.

Step 1: Define $x_\delta(t)$.

The sampled signal $x_\delta(t)$ is given as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\, \delta(t - nT_s).$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nT_s). \quad —①$$

$x(nT_s)$ is basically $x(t)$ sampled at $t = nT_s$, $n = 0, \pm 1, \pm 2$

$x(nT_s)$ = Sampled input signal.

$\delta(t-nT_s)$ = samples placed at $\pm T_s, \pm 2T_s, \pm 3T_s \ldots$

<u>Step 2</u>: FT of $x_\delta(t)$ ie $X_\delta(f)$.

Taking FT of eqn ①.

$$X_\delta(f) = FT\left\{ \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s) \right\}.$$

$$= FT\{\text{product of } x(t) \text{ and impulse train}\}.$$

The FT of product in time domain becomes convoluti in frequency domain. ie.

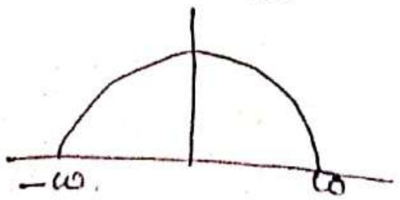
$$X_\delta(f) = FT\{x(t)\} * FT\{\delta(t-nT_s)\}. \quad — ②.$$

By definition, $x(t) \xleftrightarrow{FT} X(f)$ and

$$\delta(t-nT_s) \xleftrightarrow{FT} f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s).$$

Hence eqn ② becomes,

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s).$$



Spectrum of Original signal.x

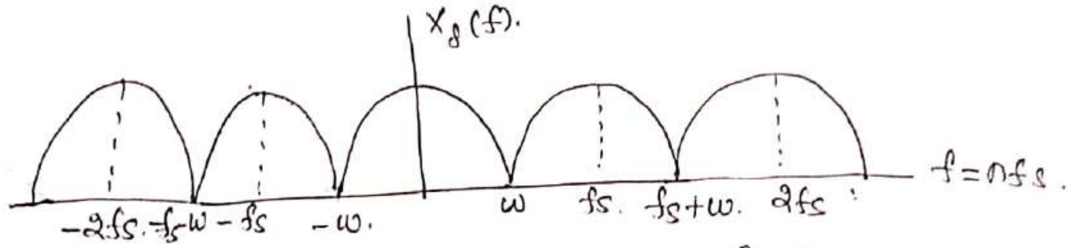
$X_\delta(f)$

Since convolution is linear.

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$ by shifting property of impulse function.

$$= f_s X(f-2f_s) + f_s X(f-f_s) + f_s X(f) +$$
$$f_s X(f-f_s) + f_s X(f-2f_s) + \cdots$$



Spectrum of sampled signal.

From the spectrum, $X(f)$ is periodic in f_s and if sampling frequency $f_s = 2W$ then the spectrum $X(f)$ just touch each other.

<u>Step 3:</u> Relation between $X(f)$ and $X_\delta(f)$.

Assume $f_s = 2W$ then

$$X_\delta(f) = f_s X(f). \quad \text{for } -W \le f \le W \text{ and } f_s = 2W.$$

$$X(f) = \frac{1}{f_s} X_\delta(f) \quad \text{— (3)}$$

<u>Step 4:</u> Relation between $x(t)$ and $x(nT_s)$.

DTFT is $$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}.$$

$$X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi fn}. \quad \text{— (4)}$$

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In above equation 'f' is the frequeny of DT signal. If we replace $X(f)$ by $X_\delta(f)$ then f becomes frequeny of CT signal ie,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi \frac{f}{f_s} n}.$$

Here f is the frequency of CT signal. And $\frac{f}{f_s}$ = frequeny of DT signal in eqn ④. Since $x(n) = x(nTs)$ ie samples of $x(t)$ then we have,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nTs)\, e^{-j2\pi f n Ts} \quad \because \frac{1}{fs} = Ts.$$

Putting above eqn in ③,

$$X(f) = \frac{1}{fs} \sum_{n=-\infty}^{\infty} x(nTs)\, e^{-j2\pi f n T_s}.$$

Inverse Fourier transform (IFT) of above eqn. gives $x(t)$ ie,

$$x(t) = IFT\left\{ \frac{1}{fs} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s} \right\} \quad -⑤$$

i) Here $x(t)$ is represented in terms of $x(nT_s)$ and.
ii) $f_s = 2w$ ie the samples are taken at the rate of $2w$ or higher since the first part of the Sampling theorem is proved by above comments.

Reconstruction of $x(t)$ from its samples:

Step 1: Take inverse fourier transform of $X(f)$ which is interms of $X_\delta(f)$

Step 2: Show that $x(t)$ is obtained back with the help of interpolation function.

Step 1:

The IFT of eqn 5 becomes,

$$x(t) = \int_{-\infty}^{\infty} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f nT_s} \Big\} e^{j2\pi ft} \cdot df.$$

Here the integration can be taken from $-w \le f \le w$ since $X(f) = \frac{1}{f_s} X_\delta(f)$ for $-w \le f \le w$.

$$\therefore x(t) = \int_{-w}^{w} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f nT_s}\, e^{j2\pi ft} \cdot df.$$

Interchanging the order of summation and integration

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \int_{-w}^{w} e^{j2\pi f(t-nT_s)}\, df.$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)}\right]_{-w}^{w}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \left\{\frac{e^{j2\pi w(t-nT_s)} - e^{-j2\pi w(t-nT_s)}}{j2\pi(t-nT_s)}\right]$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \frac{\sin 2\pi w(t-nT_s)}{\pi(t-nT_s)}.$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\pi(2Wt - 2WnT_s)}{\pi(f_s t - f_s nT_s)}$$

Here $f_s = 2W$, hence $T_s = \frac{1}{f_s} = \frac{1}{2W}$. Simplifying above eqn,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\pi(2Wt-n)}{\pi(2Wt-n)}$$

$$\because \frac{\sin\pi\theta}{\pi\theta} = \text{sinc}\,\theta$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\,\text{sinc}\,(2Wt-n). \quad — (6)$$

Step 2: Let us interpret the above eqn, Expanding we get,

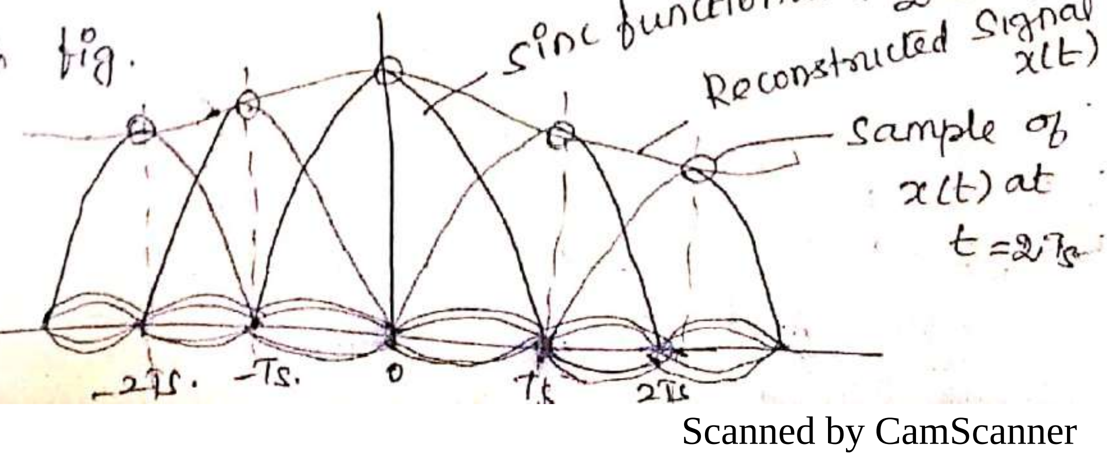
$$x(t) = \dots + x(-2T_s)\,\text{sinc}\,(2Wt+2) + x(-T_s)\,\text{sinc}\,(2Wt+1) + x(0)\,\text{sinc}\,(2Wt) + x(T_s)\,\text{sinc}\,(2Wt-1) + \dots$$

i) The samples $x(nT_s)$ are weighted by sinc functions.

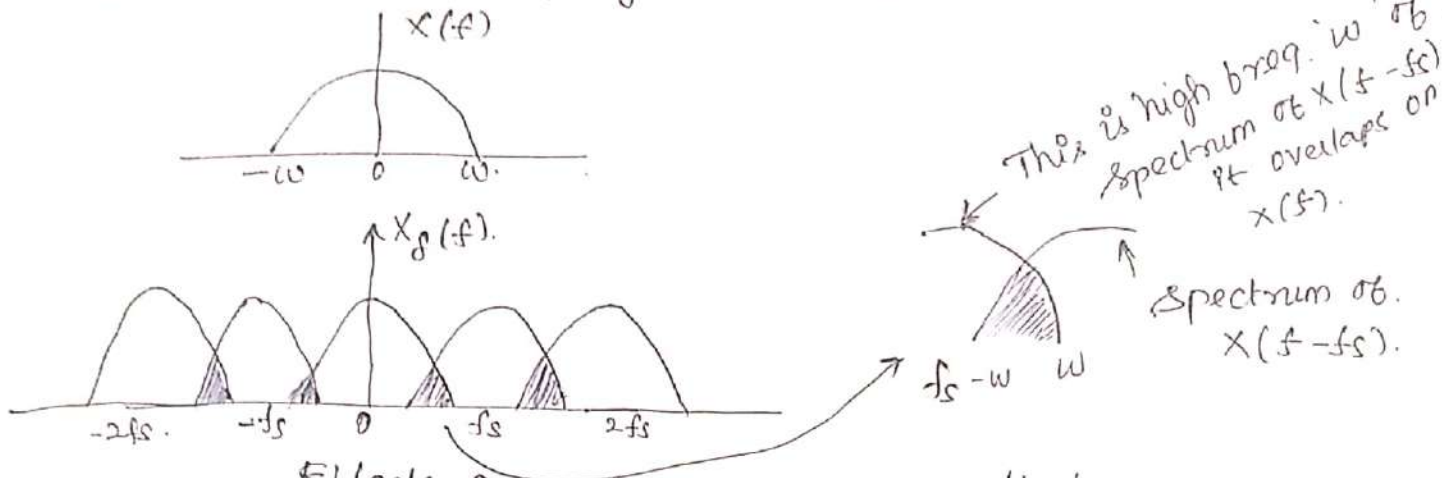
ii) The sinc function is the interpolating function.

Step 3: Reconstruction of $x(t)$ by low pass ~~[illegible]~~ filter

When the interpolated signal of eqn (6) is passed through the low pass filter of bandwidth $-W \le f \le W$ then the reconstructed waveform shown in fig.



Effects of undersampling [Aliasing]:



Effects of undersampling (or) aliasing.

The spectrum located at $X(f)$, $X(f-f_s)$, $X(f-2f_s)$.... overlap on each other.

Consider the spectrum of $X(f)$ and $X(f-f_s)$, the frequencies from $(f_s-\omega)$ to ω are overlapping in these spectrums.

The high frequencies near 'ω' in $X(f-f_s)$ overlap with low frequencies $(f_s-\omega)$ in $X(f)$.

Aliasing :-

when the high frequency interferes with low frequency and appears as low frequency, then the phenomenon is called aliasing.

Effects of aliasing :

The high and low frequencies interfere with each other, distortion is generated.

The data is lost and it cannot be recovered.

Different ways to avoid aliasing:

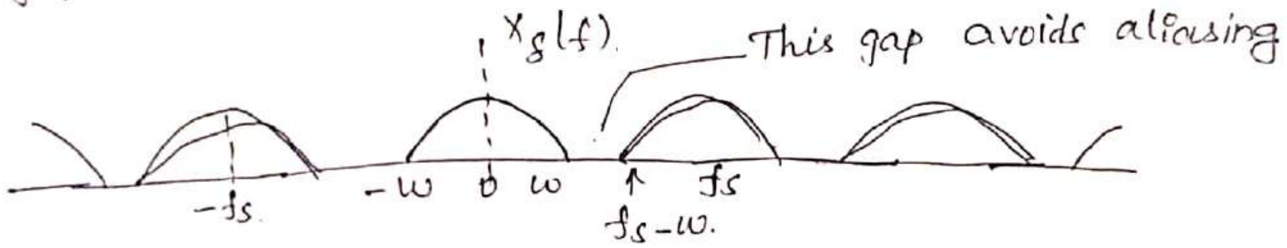
Aliasing can be avoided by two methods.

i) Sampling rate $f_s \geq 2W$.

ii) Strictly bandlimit the signal to 'W'.

i) Sampling rate $f_s \geq 2W$.

When the sampling rate is higher than $2W$, then the spectrum will not overlap and there will be sufficient gap between the individual spectrum.



Oversampling:

When the signal is sampled at a rate much higher than Nyquist rate, it is called Oversampling. It increases transmission bandwidth.

ii) Bandlimiting the signal:

The sampling rate $f_s = 2W$, ideally there is no aliasing effect but practically a low pass filter is used before sampling the signals. Thus the output of low pass filter is strictly bandlimited and there is no frequency component higher than W. Then there will be no aliasing.

$x'(t) \longrightarrow$ [Band limiting Low pass filter] $\xrightarrow{x(t)}$ [Sampler] $\longrightarrow x_\delta(t)$.

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Nyquist rate and Nyquist Interval:

Nyquist rate:

when the sampling rate becomes exactly equal to 2W samples/sec. for a given bandwidth of 'W' Hertz, then it is called Nyquist rate.

$$\text{Nyquist rate} = 2W \text{ Hz.}$$

Nyquist interval:

It is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

$$\text{Nyquist interval} = \frac{1}{2W} \text{ sec.}$$

Reconstruction filter [Interpolation filter]:

Reconstructed signal is the succession of Sinc pulses weighted by $x(nT_s)$. These pulses are interpolated with the help of low pass filter. It is also called reconstruction filter (or) interpolation filter.

Problems:

1) The signal $g(t) = 10 \cos(40\pi t) \cos(400\pi t)$ is sampled at the rate of 500 samples/sec.

i) Determine the Nyquist rate.

ii) Calculate the cut-off frequency of ideal reconstruction filter.

iii) Draw the spectrum of resulting sampled signal.

iv) If $g(t)$ is considered to be a band pass signal, determine the lowest permissible sampling rate.

Solution:

$$g(t) = 10 \cos(40\pi t) \cos(400\pi t).$$

$$= \frac{10}{2}\left[\cos[40\pi t - 400\pi t) + \cos(40\pi t + 400\pi t]\right].$$

$\cos(A - B) + \cos(A + B)./2.$

$$= 5\cos(360\pi t) + 5\cos(440\pi t)$$

Comparing this signal with $g(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$

$A_1 = 5V$ and $f_1 = 180\,Hz$ $A_2 = 5V$ and $f_2 = 220\,Hz$.

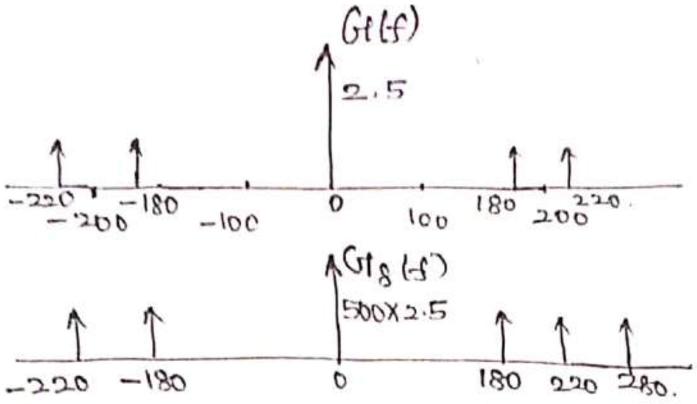
Thus the highest signal frequency is $W = f_2 = 220\,Hz$.

i) Nyquist rate $= 2W = 2 \times 220 = 440$ samples/sec.

ii) Cut-off frequency $f_c = W = 220\,Hz$.

iii) Spectrum of sampled signal.

here $f_s = 500\,Hz$. The frequency component of cosine waves are placed at $\pm f_1$, and $\pm f_2$. Their amplitudes are reduced by half.



Spectrum of sampled signal.

iv) Sampling rate for bandpass & signal consideration.

Here the bandwidth of the signal is

$B_x = f_2 - f_1$

$= 220 - 180 = 40$ Hz.

Sampling rate $= 2B_x = 2 \times 40 = 80$ Hz.

2) Determine the Nyquist rate and interval for the following signal.

$x(t) = 5\cos(2000t) + 7\sin(7000t)$.

Solution:

compare the signal with.

$x(t) = 5\cos(\omega_1 t) + 7\sin(\omega_2 t)$.

$f_1 = \frac{2000}{2\pi} = 318.3$ Hz $\qquad f_2 = \frac{7000}{2\pi} = 1114.1$ Hz.

The highest freq is $w = 1114.1$ Hz.

Nyquist rate $= 2W = 2 \times 1114.1 = 2228.17$ Hz.

Nyquist interval $= \frac{1}{2W} = 4.488 \times 10^{-4}$ sec.

③ A waveform $g(t) = 20+20 \sin(500\pi t + 30°)$ is to be sampled periodically and reproduced from the samples.

i) Find the max. allowable time interval between the samples

ii) How many sample values are need to be stored in order to produce one second of this waveform, if sampled according to the result in (i).

Solution:

$$g(t) = 20 + 20 \sin[500\pi t + 30°].$$

Here max. freq of the signal is $W = 250\,Hz$

(i) To obtain max. allowable time interval between the samples.

Nyquist rate $= 2W = 2 \times 250 = 500\,Hz.$

Max. time. interval $= \frac{1}{2W} = \frac{1}{500} = 2\,ms.$

(ii) To obtain number of sample values.

The sampling rate as per (i) is 500Hz. This means 500 samples per second. Hence 500 samples are need to be stored in order to produce one second of the waveform.

QUANTISATION

✓ The Conversion of analog signal to digital Signal (ie) it gives a discretized amplitude level. The input sample value is Quantized to nearest digital level.

This Quantisation can be,

a) UNIFORM (OR) LINEAR QUANTISATION

b) NON UNIFORM (OR) NON-LINEAR QUANTISATION

✓ In uniform Quantization, the step size (or) difference between two Quantization levels remains Constant over the Complete amplitude range.

✓ In non-uniform quantization, the step size is not fixed, it varies accordance to Certain law (or) as per i/p sgl. amplitude.

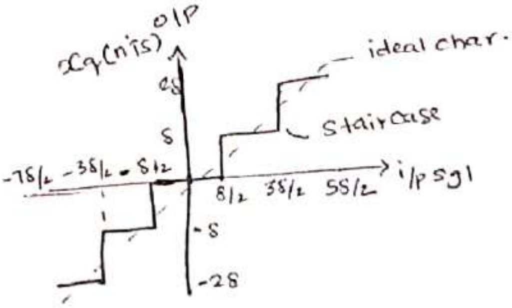
Depends up on the transfer Characteristics, there are 3 types of uniform (or) linear Quantizers,

UNIFORM QUANTISATION:

$\left.\begin{array}{l}\rightarrow \text{Midtread Quantiser,} \\ \rightarrow \text{Midrise Quantiser}\end{array}\right\}$ Rounding

→ Biased Quantiser → Truncation

Mid tread Quantizer



When i/p between $-\delta/2$ & $+\delta/2$ then o/p = 0;

(ie) $-\delta/2 \le x(nTs) < \delta/2$; $x_q(nTs) = 0$

for,

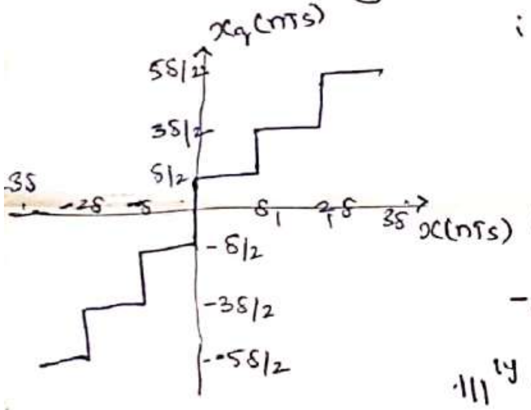
$\delta/2 \le x(nTs) < 3\delta/2$; $x_q(nTs) = \delta$

Quantization Error $(\epsilon) = x_q(nTs) - x(nTs)$

$$-\delta/2 \le \epsilon \le \delta/2$$

$$\boxed{\epsilon_{max} = |\delta/2|}$$

Midrise Quantizer:



i/p b/w 0 & δ, o/p $\rightarrow \delta/2$

III^{ly} 0 & $-\delta$; o/p $\rightarrow -\delta/2$

$0 \le x(nTs) < \delta$; $x_q(nTs) = \delta/2$

$-\delta \le x(nTs) < 0$; $x_q(nTs) = -\delta/2$

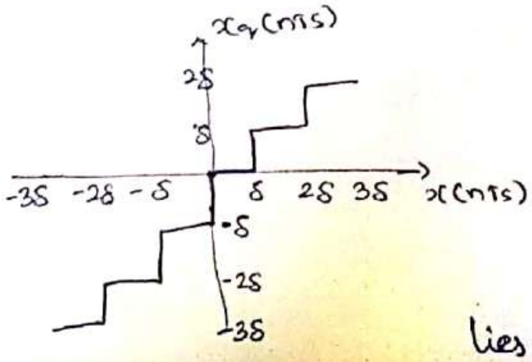
III^{ly} i/p b/w 3δ & 4δ o/p $\rightarrow 7\delta/2$

$\epsilon \rightarrow$ i/p 0; $+\delta/2$

$$\epsilon = x_q(nTs) - x(nTs)$$
$$= \delta/2 - 0$$
$$\boxed{\epsilon = \delta/2}$$

$$\boxed{\epsilon_{max} = |\delta/2|}$$

Biased Quantizer:



i/p 0 & $\delta \rightarrow$ o/p $\rightarrow 0$

$0 \le x(nTs) < \delta$; $x_q(nTs) = 0$

$-\delta \le x(nTs) < 0$; $x_q(nTs) = -\delta$

i/p $\delta \rightarrow$ o/p $\rightarrow 0$

$\epsilon = 0 - \delta$

$$\boxed{\epsilon_{max} = |\delta|}$$

lies b/w 0 & $-\delta$; $-\delta \le \epsilon \le 0$

DERIVATION OF MAXIMUM SIGNAL TO QUANTIZATION NOISE RATIO FOR LINEAR QUANTIZATION

May June 2013
Nov-dec 2013.

(OR)

SNR FOR UNIFORM QUANTIZATION

Step 1: Quantization Error: Because of quantization, inherent error are introduced in sgl,

$$\boxed{\epsilon_q = x_q(nT_s) - x(nT_s)}$$

Step 2: Step size (δ)

I/p $x(nT_s)$ be continuous amplitude in range $-x_{max}$ to $+x_{max}$

Total amplitude range $= x_{max} - (-x_{max}) = 2x_{max}$

Amplitude range is divided in to 'q' levels of quantizer, then,

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

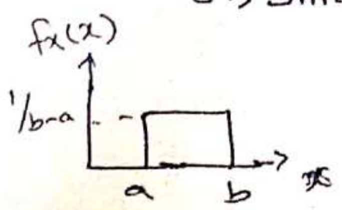
$$\boxed{\delta = \frac{2x_{max}}{q}}$$

$x(t)$ is normalized to min to max $= 1$

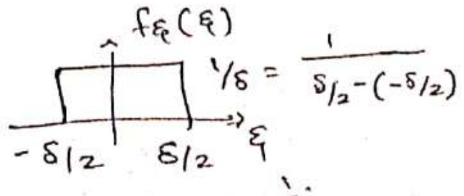
$$\therefore \boxed{\delta = 2/q}$$

Step 3: Pdf of 'ϵ_q' $\therefore \epsilon_{q\,max} = |\delta/2|$

$\delta \to$ small, 'ϵ_q' uniformly distributed,



Unif. dst.



Uni dst. for ϵ_q

$$f_x(x) = \begin{cases} 0 & x \le a \\ 1/b-a & a < x \le b \\ 0 & x > b \end{cases}$$

pdf. of ε

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & ; \varepsilon \le -\delta/2 \\ \frac{1}{\delta} & ; -\delta/2 < \varepsilon \le \delta/2 \\ 0 & ; \varepsilon > \delta/2 \end{cases}$$

Step 4 SNR

$$S/N = \frac{\text{Sgl. Power}}{\text{Noise power}}$$

Noise power, $= \frac{V^2_{Noise}}{R}$; $\longrightarrow$ mean square value of noise Vge

$$E(\varepsilon^2) = \overline{\varepsilon}^2$$

$$\overline{X}^2 = E(x^2) = \int_{-\infty}^{\infty} x^2 f_x(x) \cdot dx$$

Here,

$$E(\varepsilon^2) = \int_{-\infty}^{\infty} \varepsilon^2 \cdot f_\varepsilon(\varepsilon) \cdot d\varepsilon.$$

$$E[\varepsilon^2] = \int_{-\delta/2}^{\delta/2} \varepsilon^2 \cdot \frac{1}{\delta} \cdot d\varepsilon$$

$$E[\varepsilon^2] = \frac{1}{\delta} \left[\frac{\varepsilon^3}{3}\right]_{-\delta/2}^{\delta/2}$$

$$E[\varepsilon^2] = \delta^2/12 \quad ; R=1$$

$\therefore$ Noise Power $\boxed{V^2_{noise} = \frac{\delta^2}{12}}$

No. of bits 'v' & quantization level 'q' relates

$$\boxed{q = 2^{v}}$$

$$\therefore \delta = \frac{2x_{max}}{2^{v}}$$

$$\frac{S}{N} = \frac{P}{\left(\frac{2x_{max}}{2^{v}}\right)^2 / 12} = \frac{P}{\frac{4x^2_{max}}{2^{2v}} \times \frac{1}{\cancel{12}\,3}} = \frac{3P}{x^2_{max}} 2^{2v}$$

$$\boxed{\frac{S}{N} = \frac{3P}{x^2_{max}} \cdot 2^{2v}}$$

$P \leq 1$; $x_{max} = 1$

$$\frac{S}{N} \leq 3 \times 2^{2v}$$

$$\left(\frac{S}{N}\right)_{db} = 10 \log_{10} \left(\frac{S}{N}\right) dB$$

$$\leq 10 \log_{10} \left[3 \times 2^{2v}\right]$$

$$\boxed{\frac{S}{N} \leq (4.8 + 6v)\, dB}$$ //

NON-UNIFORM QUANTIZATION (Apr-may 2011)

(Acheired through 'Companding) (Apr-may 2010)

→ Step size is small at low i/p sgl level, hence (ϵ) → Small ; ∴ (S/N → improved at low sgl. level)

→ Step size is higher at high i/p sgl level. (S/N) → remains almost same throughout the dynamic range of quantizer.

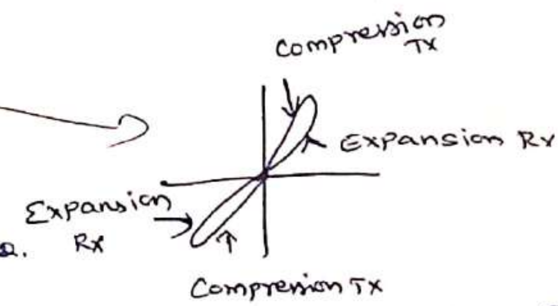
NECESSITY OF NON UNIFORM QUANTIZATION:

Speech and music signal are characterised by large crest factor.

$$\text{Crest factor} = \frac{\text{Peak Value}}{\text{RMS Value}} = \frac{x_{max}}{x^2(t)}$$

$$P = \frac{V^2_{sgl}}{R} = \frac{x^2(t)}{R} \quad \therefore \boxed{C\cdot F = 1/\sqrt{P}}$$

Companding in PCM



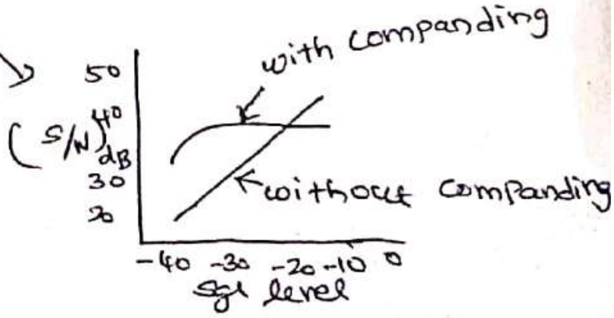
Companding Curves.

μ law Compression, used.

Speech & music sgl. The Compression is defined by foll. eq.

$$Z(x) = \text{sgn}(x) \cdot \frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \; ; \; |x| \le 1$$

'A' law Companding



$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} \; ; & 0 \le |x| \le 1/A \\ \dfrac{1+\ln(A|x|)}{1+\ln A} & 1/A \le |x| \le 1 \end{cases}$$

Practical value of $A = 87.56$

A & μ law used in PCM telephone system

SNR of Companded PCM $= \frac{3q^2}{[\ln(1+\mu)]^2}$

Non-Uniform Quantization and Logarithmic companding:

Non-Uniform Quantization for Speech signal:

In non-uniform quantization, the step size is not fixed. It varies according to certain conditions as per input signal amplitude.

The speech and music signals are characterized by large crest factor. That is for such signals the ratio of peak to rms value is very high.

$$\text{Crest factor} = \frac{\text{Peak value}}{\text{RMS value}} \quad -(1)$$

$$\frac{S}{N} = (3 \times 2^{2v} \times P).$$

Expressing in decibels $\left(\frac{S}{N}\right)_{dB} = 10\log_{10}(3 \times 2^{2v} \times P).$

If we normalize the signal power ie if $P=1$, then above equation becomes,

$$\left[\frac{S}{N}\right]_{dB} \geq [4.8 + 6v)\, dB. \quad -(2)$$

Here p is defined as, $P = \frac{V^2}{R} = \frac{x^2(t)}{R}. \quad -(3)$

V^2_{signal} = Mean square value of signal Voltage $= x^2(t)$.

Normalized power will be $P = \frac{x^2(t)}{1}$. $\because R = 1$.

$$P = x^2(t)$$

$$\text{Crest factor} = \frac{\text{Peak value}}{\text{RMS Value}} = \frac{x_{max}}{[x^2(t)]^{1/2}} \quad -⑤.$$

$$= \frac{x_{max}}{\sqrt{P}} \quad \because P = x^2(t). \quad -⑥.$$

when we normalise the signal $x(t)$, then

$$x_{max} = 1. \quad -⑦$$

Putting the above value of x_{max} in eqn ⑥.

$$\text{crest factor} = \frac{1}{\sqrt{P}}. \quad -⑧.$$

For a large crest factor of voice and music signal P should be very very less than one in above equation.

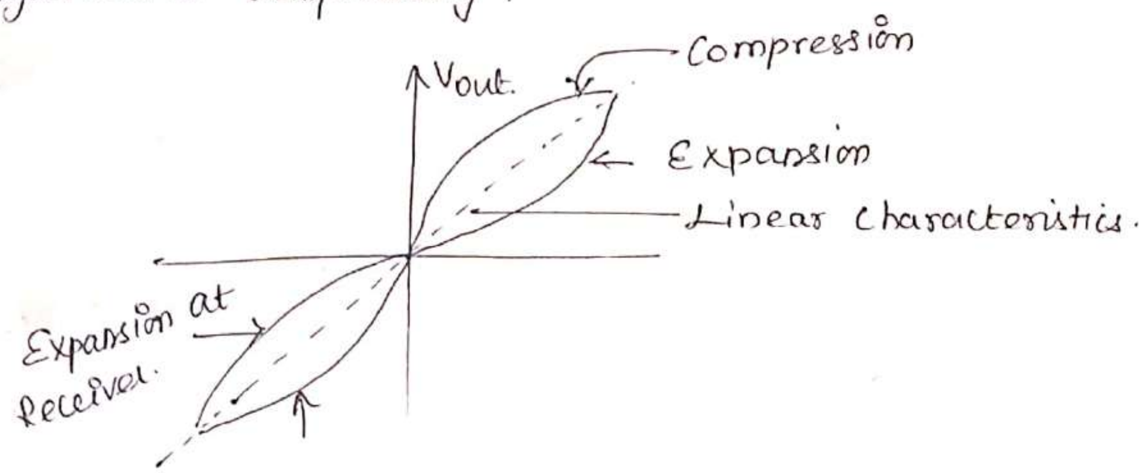
$$P << 1.$$

Therefore actual signal to noise ratio will be significantly less than the value that is given by eqn③ since in this equation $P=1$. consider equation ②,

$$\left[\frac{S}{N}\right] = 3 \times 2^{2v} \times P.$$

$$[3 \times 2^{2v} \times P)\Big|_{P<<1} << (3 \times 2^{2v} \times P)\Big|_{P=1} \quad -⑨$$

Logarithmic Companding :

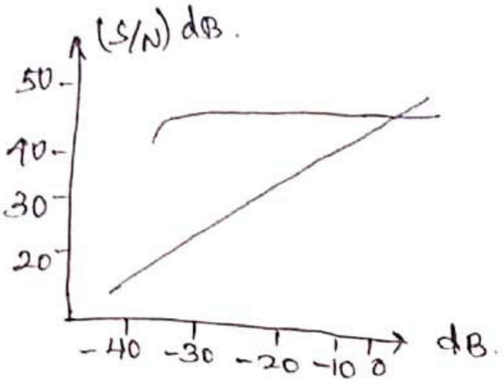


Compression at transmitter.

The non uniform quantization becomes difficult to implement because of variable step size δ. The signal is amplified at low signal levels and attenuated at high signal levels. After this process, uniform quantization is used. This is equivalent to more step size at low signal levels and small step size at higher signal levels.

At the receiver a reverse process is done. That is signal is attenuated at low signal level and amplified at high signal level to get original signal. Thus the compression of signal at transmitter and expansion at receiver is called companding.

μ-law Companding for speech signal:



The speech & music signals expressed by μ law companding.

$$\boxed{z(x) = (\text{sgn}\, x) \frac{\ln[1+\mu|x|]}{\ln(1+\mu)}}$$

$|x| \leq 1.$

A law companding:

A law companding provides compressor characteristics. It has linear segment for low level inputs and logarithmic segment for high level inputs.

$$z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \leq |x| \leq \dfrac{1}{A} \\ \dfrac{1+\ln[A|x|]}{1+\ln A} & \text{for } \dfrac{1}{A} \leq |x| \leq 1. \end{cases}$$

when $A=1$, we get uniform quantization. The practical value of A is 87.56. Both A law & μ law companding is used for PCM telephone system.

Signal to Noise Ratio of Companded PCM :

(i) SNR for μ-law companding.

$$\frac{S}{N} = \frac{3q^2}{[\ln(1+\mu)]^2}$$

here $q = 2^v$ is no. of quantization levels.

(ii) Signal to noise for A law companding

$$\left[\frac{S}{N}\right] = \frac{3q^2 \cdot A^2}{(1+\ln A)^2}$$

Here $q = 2^v$ are the no. of. quantization levels.

In the above 2 equations, the companding Gain G_c in μ law and A law is given as.

for μ law companding gain $G_c = \dfrac{\mu}{\ln(1+\mu)}$

for A law companding gain $G_c = \dfrac{A}{1+\ln A}$

Pulse analog Modulation:

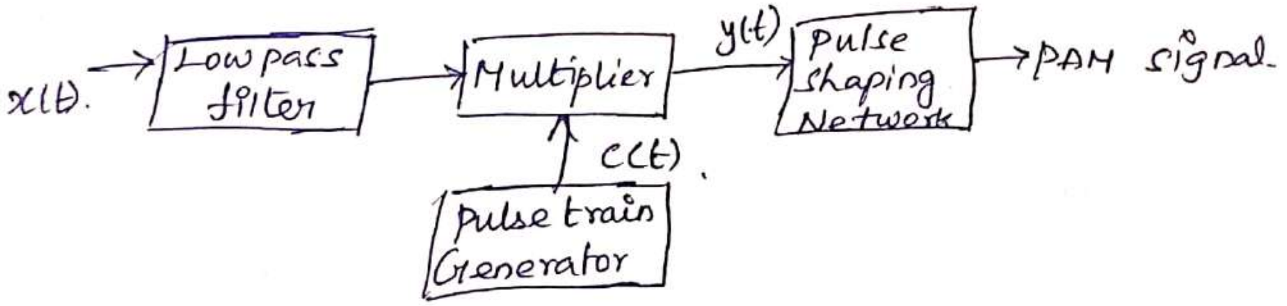
The modulating signal can modulate amplitude, width or position of the pulse. Depending upon this, three techniques are possible.

i) PAM ii) PWM and iii) PPM.

Pulse Amplitude Modulation (PAM):

The amplitude of the pulse is directly proportional to amplitude of the modulating signal at the sampling instant. The width and position of the pulse remains unchanged.

Generation of PAM:



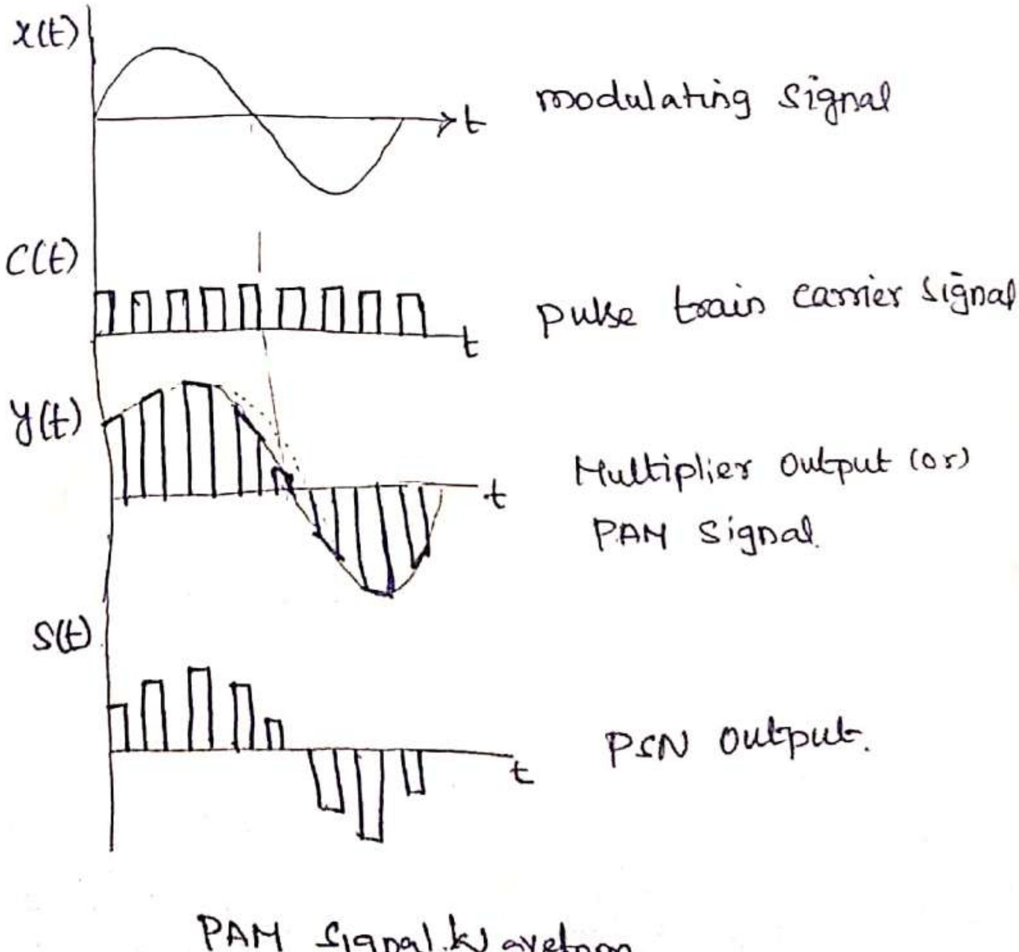
The modulating signal x(t) is passes to lowpass filter and it performs bandlimiting. The cut off frequency of the low pass filter is equal to highest freq (fm) present in x(t).

Lowpass filtering avoids aliasing.

The bandlimited signal is then sampled at the multiplier. The multiplier samples $x(t)$ with the help of a pulse train generator. The pulse train generator produces the pulse train $c(t)$. The multiplication of $x(t)$ & $c(t)$ produces the PAM signal $y(t)$.

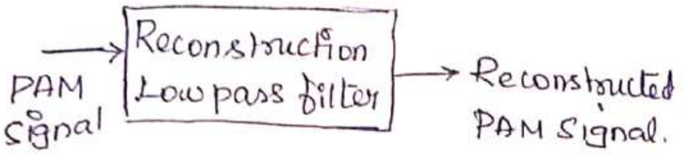
The top of the pulses are varied according to the amplitude of $x(t)$.

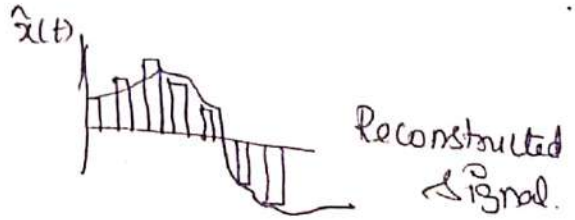
The pulse shaping network produces the flat top pulses.



PAM signal waveform.

Detection of PAM :-





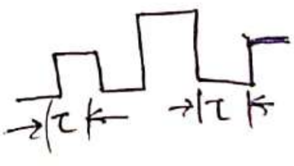
The PAM signal is passed through a low pass filter, it reconstructs the analog signal from PAM pulses.

Transmission bandwidth of PAM signal:

The pulse duration τ is supposed to be very very small compared to time period T_s between the 2 samples. If the max. frequency in the signal $x(t)$ is W then by sampling theorem f_s should be higher than Nyquist rate or,

$$f_s \geq 2W \text{ or}$$

$$T_s \leq \frac{1}{2W}.$$



If ON and OFF time of the pulse is same, then freq of the PAM pulse becomes,

$$f = \frac{1}{\tau+\tau} = \frac{1}{2\tau}.$$

Bandwidth of PAM will be equal to max. frequency of the signal.

$$B_T \geq f_{max}.$$

$$B_T \geq \frac{1}{2\tau}.$$

since $\tau << \frac{1}{2W} \geq \frac{1}{2\tau} >> W.$ $\boxed{\because B_T >> W}$

Pulse width Modulation [PWM]:

There are two types of pulse time modulation [PTM].

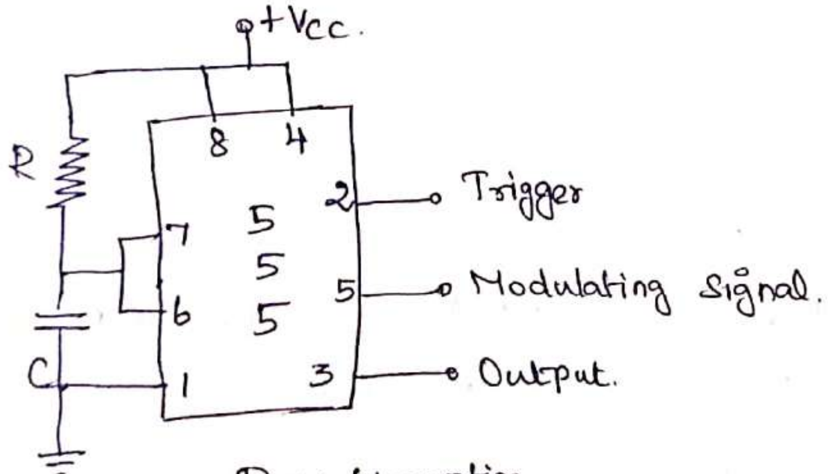
i) Pulse width modulation (or) pulse Duration Modulation (PWM) (or) (PDM).

ii) pulse position modulation (PPM).

Principle of PWM (or) PDM:

The width of the pulse is directly proportional to amplitude of the modulating signal at the sampling instant. The amplitude & position of the pulse remains unchanged.

PWM Generation:

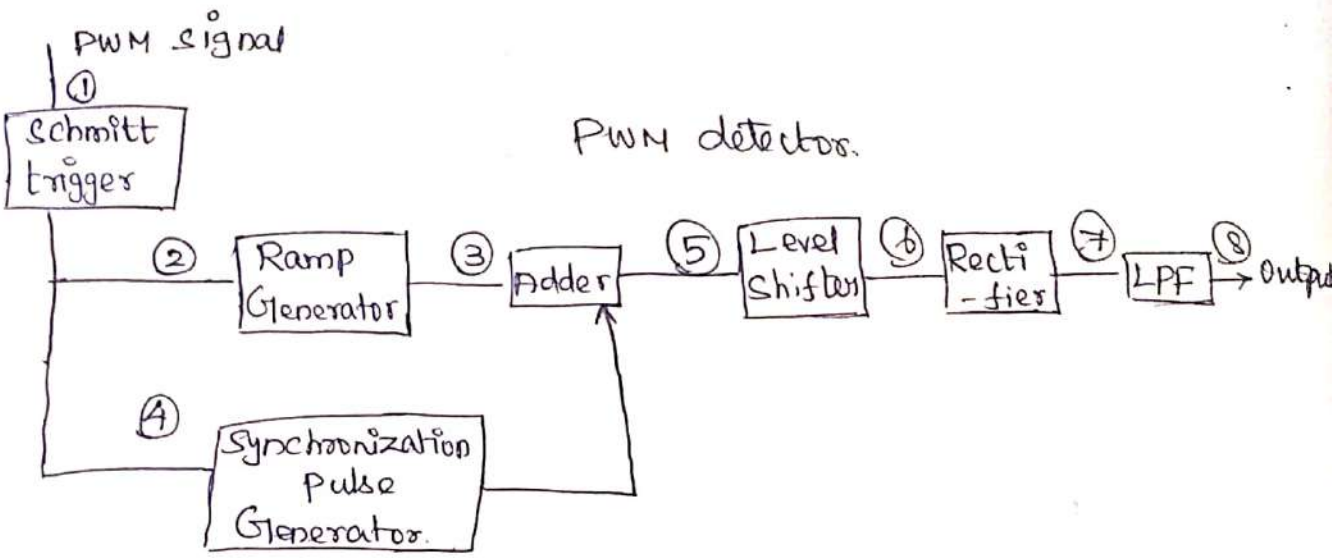


PWM Generation.

The circuit is basically a monostable multivibrator with a modulating input signal applied at the Control Voltage input.

Internally, the control voltage is adjusted to the $2/3$ Vcc. Externally applied modulating signal changes the control voltage and threshold voltage. As a result, time period required to charge the capacitor up to threshold voltage level changes, giving pulse modulated signal at the output.

Demodulation of PWM :



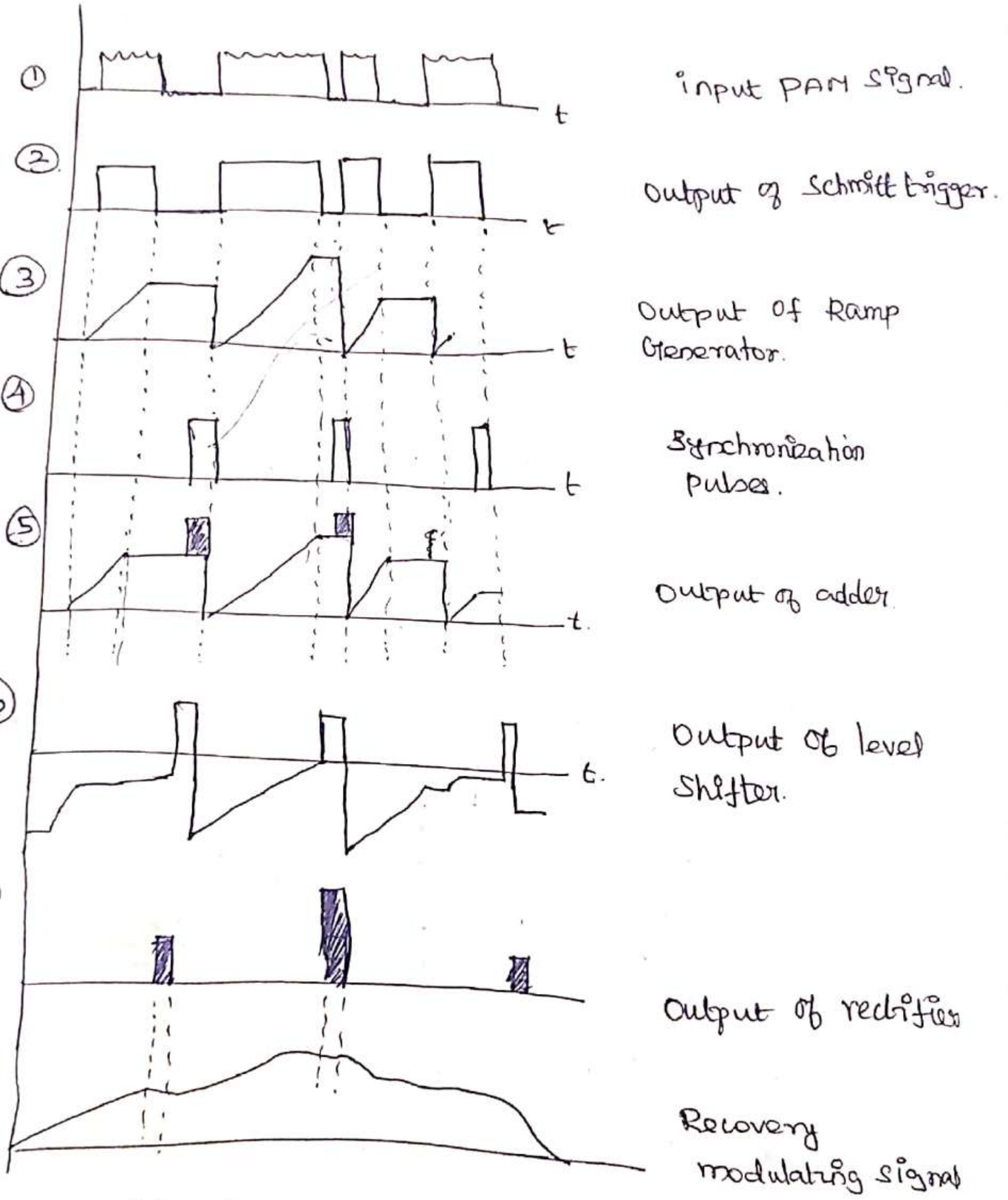
1. The PWM signal is applied to Schmitt trigger it removes the noise and the regenerated PWM is applied to the ramp generator and synch. pulse Generator.

2. The ramp generator produces ramps for the duration of pulses such that height of the ramps are proportional to the width of PWM signal.

3. The synchronous pulse generator produces reference pulses with constant amplitude and pulse width. These pulses are delayed by specific ~~constant~~ amount of delay.

4. The delayed reference pulses and the output of ramp generator is added with the help of adder. Level shifter shift the level of the signal and the negative offset waveform is clipped by rectifier.

The output of rectifier is passed through Low pass filter to recover the modulating signal.

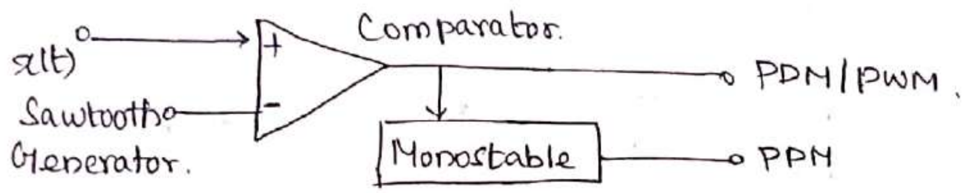


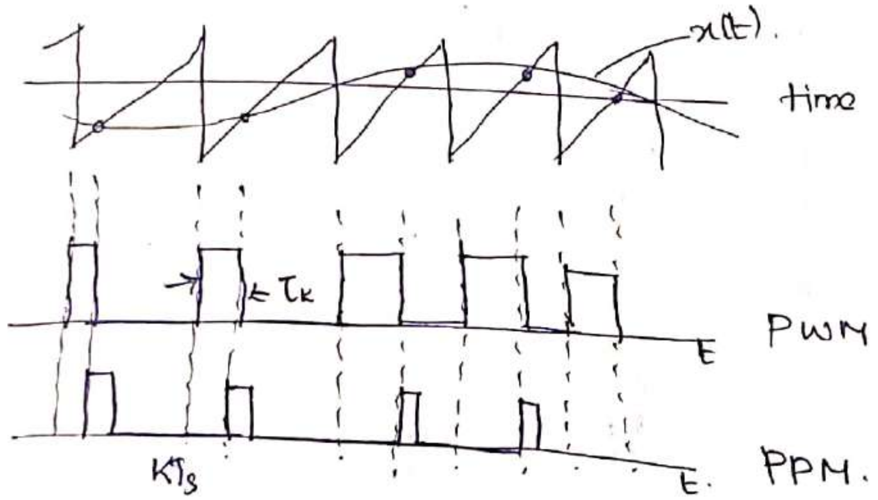
Waveform of PWM detection circuit.

Pulse position Modulation [PPM]:

The amplitude and width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse is changed in accordance with the modulating signal.

Generation of PPM:





The sawtooth signal or sampling signal is applied to the inverting input of Comparator. The modulating signal $x(t)$ is applied to the non inverting input of the Comparator.

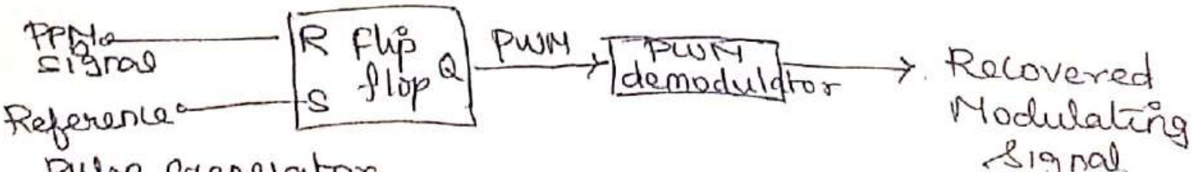
The output of the comparator is high only when instantaneous value of $x(t)$ is higher than that of sawtooth waveform.

Thus the leading edge of ~~output~~ PPM signal occurs at the fixed time period ie KT_s the trailing edge of output of Comparator, depends on the amplitude of signal $x(t)$. when sawtooth waveform Voltage is greater than Voltage of $x(t)$, the output of comparator remains Zero.

The trailing edge of the output of Comparator is modulated by the signal $x(t)$. If the sawtooth waveform is reversed, then trailing edge will be fixed and leading edge will be modulated.

PPM signal is generated by monostable multivibrator. PWM signal is used to trigger the one monostable multivibrator. It is triggered on the falling (trailing) edge of PDM. The output of monostable then switches to positive saturation level. This Voltage remains high for the fixed period then goes low. The width of the pulse can be determined by monostable.

Demodulation of PPM Signal:



The flip flop circuit is set or turned 'ON' when the reference pulse arrives. This reference pulse is generated by pulse generator.

The flipflop is reset or turned 'Off' at the leading edge of the position modulated pulse. This repeats and we get PWM at the output of the flip flop. The demodulator of PWM is used to recover the modulating signal.

Transmission bandwidth of PPM and PDM.

The rise time should be very less than T_s. ie

$t_r << T_s$. and

transmission bandwidth should be,

$$\boxed{B_T \geq \frac{1}{2t_r}}$$ (for PDM & PPM).

Time division Multiplexing:-

The signals from different channels are transmitted over a common transmission line. The time spacing between the samples of one channel is occupied by other channel to improve the channel utilization. This is called TDM.