GRT INSTITUTE OF ENGINEERING AND TECHNOLOGY

TIRUTTANI

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC8561 – Transmission Lines and RF Systems

Course Material

Unit – I

TRANSMISSION LINE THEORY

Find the reflection coefficient of a 50-ohm transmission line when it is terminated by a load impedance of 60 + j40 ohm.

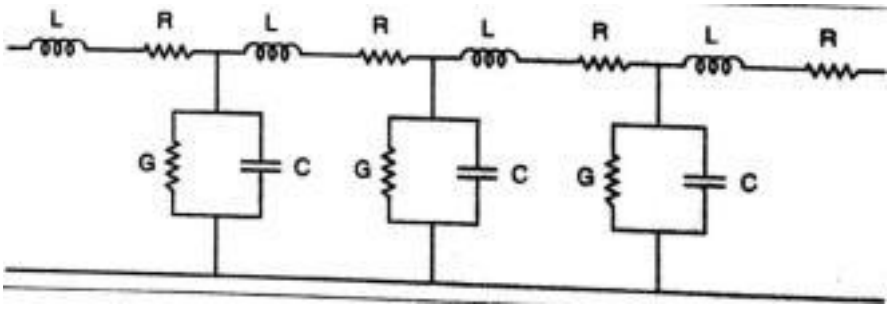
=

=

50 – 60 – 40 +

K = 0.35 124.01

Equivalent circuit of a unit length of a transmission line:



Infinite line:

When S=, in the infinite line the travelling waves continue in one direction indefinitely and there is no source of energy or discontinuity to send back a reflected wave along the line.

Delay distortion:

For an applied voice-voltage wave the received waveform may not be identical with the input waveform at the sending end, since some frequency components will be delayed more than those of other frequencies. This phenomenon is known as delay or phase distortion.

$$\text{Reflection Factor } k = \left|\frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0}\right| = \left|\frac{2\sqrt{745\times 100}}{745+100}\right| = 0.645.$$

$$\text{Reflection Loss} = 20\log\frac{1}{|k|} = 20\log\frac{1}{0.645} = 3.7751 \text{ dB}$$

Unit – II

HIGH FREQUENCY TRANSMISSION LINES

Input impedance of open and short - circuited dissipation less line:

Short circuited impedance

$$Z_{sc} = Z_o \tanh \gamma l$$

The open circuited impedance

$$Z_{oc} = Z_o \coth \gamma l$$

State the assumptions for the analysis of the performance of the radio frequency line:

1) Due to the skin effect, the currents are assumed to flow on the surface of the conductor. The internal inductance is zero. 2) The resistance R increases with f while inductance L increases with f. Hence ωL >> R. 3) The leakage conductance G is zero.

Standing wave ratio:

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves called standing wave ratio. $S = \frac{E_{max}}{E_{min}} = \frac{|I_{max}|}{|I_{min}|}$

Input impedance of a dissipation less line:

The input impedance of a dissipation less line is given by, $Z_s = \frac{E_s}{I_s} = R_o \frac{1 + k - 2 s}{1 - k - 2 s}$

Range of values of standing wave ratio:

The range of values of standing wave ratio is theoretically 1 to infinity.

Relation between SWR and reflection coefficient:

$S = \frac{1+|k|}{1-|k|}$, Also $|K| = \frac{s-1}{s+1}$

Unit – III
IMPEDANCE MATCHING IN HIGH FREQUENCY LINES

Use of eighth wave line:

An eighth wave line is used to transform any resistance to an impedance with a magnitude equal to Ro of the line or to obtain a magnitude match between a resistance of any value and a source of Ro internal resistance.

Input impedance of eighth wave line:

The input impedance of eighth wave line terminated in a pure resistance R_r. Is given by $Z_s = (Z_R + jR_o) / (R_o + j Z_R)$. From the equation it is seen that $|Z_s| = R_o$.

Impedance inverter:

A quarter wave line may be considered as an impedance inverter because it can transform low impedance into high impedance and vice versa.

Copper insulator:

An application of the short -circuited quarter wave line is an insulator to support an open wire line or the center conductor of a coaxial line. This application makes some of the fact that the input impedance of a quarter wave line is very high, such lines are sometimes referred to as copper insulators.

Double stub matching is preferred over single stub matching:

Double stub matching is preferred over single stub due to following disadvantages of single stub.

1. Single stub matching is useful for a fixed frequency. So as frequency changes the location of single stub will have to be changed.
2. The single stub matching system is based on the measurement of voltage minimum; hence for coaxial line it is very difficult to get such voltage minimum, without using slotted line section.

1. Design a quarter wave transformer to match a load of 200Ω to a source resistance of 500 Ω. The operating frequency is 200 MHz.

$$R_o = \sqrt{Z_s Z_R} = \sqrt{500 x 200} = 316.22\ \Omega.$$

$\lambda = C / f = 1.5$ m $\qquad\qquad \lambda / 4 = 0.375$ m.

Unit – IV
WAVE GUIDES

TEM wave or principal wave:

TEM wave is a special type of TM wave in which an electric field E along the direction of propagation is also zero. The Tem waves are waves in which both electric and magnetic fields are transverse entirely but have no components of Ez and Hz. It is also referred to as the principal wave.

Characteristics of TEM waves:

a) It is a special type of TM wave.
b) It doesn't have either E or H component.
c) Its velocity is independent of frequency.
d) Its cot-off frequency is zero.

Dominant mode for the rectangular waveguide:

The lowest mode for TE wave is TE_{10} (m=1, n=0) whereas the lowest mode for TM wave is TM_{11} (m=1, n=1). The TE_{10} wave has the lowest cutoff frequency compared to the TM_{11} mode. Hence the TE_{10} (m=1, n=0) is the dominant mode of a rectangular waveguide. Because the TE_{10} mode has the lowest attenuation of all modes in a rectangular waveguide and its electric field is definitely polarized in one direction everywhere.

A rectangular has the following dimensions l = 2.54 cm, b = 1.27 cm. Waveguide thickness = 0.127 cm. Calculate the cut off frequency for TE_{11} mode:

A rectangular waveguide has the following dimensions:
a = 2.54 cm, b = 1.27 cm, Waveguide thickness = 0.127 cm.

$$f_c = \frac{1}{2\sqrt{\ }}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 16.15\text{ GHz.}$$

$a = 2.54\text{x}10^{-2} \text{ x } 0.127 = 0/02286$ m $\qquad b = 1.27\text{x}10^{-2} \text{ x } 0.127 = 0.01016$ m.

Quality factor of a resonator:

The quality factor Q is a measure of frequency selectivity of the resonator. It is defined as Q = 2 x Maximum energy stored / Energy dissipated per cycle = W / P. Where W is the maximum stored energy, P is the average power loss.

Unit – V

RF SYSTEM DESIGN CONCEPTS Comparison of conditional and unconditional stabilities of an amplifier:

Conditional stabilities	unconditional stabilities
Conditional stabilities refers to a network that is stable when its input and output see the intended characteristic impedance Zo	Unconditional stabilities refers to a network that can see any possible impedance on the smith chart from the center to the perimeter at any phase angle. Gamma<1 means that the real part of the impedance is positive
If there is a mismatch, there is a region of either source or load impedances that will definitely cause it to oscillate. The term potentially unstable refers to the same condition	Note that any network can oscillate if it sees a real impedance that is negative, so if your system goes outside the normal smith chart all bets on stability are off

1. Power gain of amplifier:

$$G_{TU} = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-\Gamma_L S_{22}|^2|1-S_{11}\Gamma_S|^2}$$

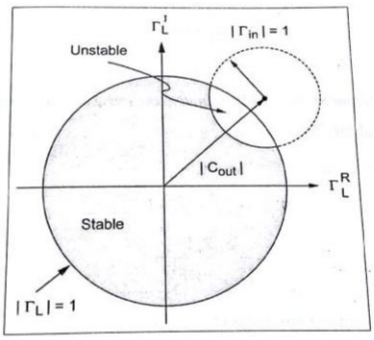
Requirement of impedance matching:

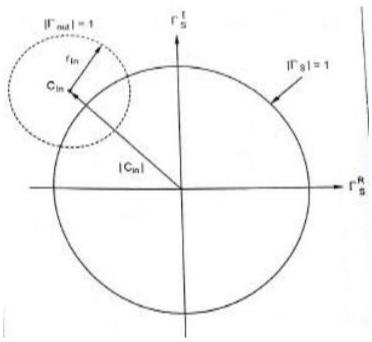
1. Minimum power loss in the feed line.
2. Maximum power transfer
3. Improving the S/N ratio of the system for sensitive receiver components

Required Other considerations:

1. Complexity 2. Band width requirement 3. Adjustability
4. Implementation

Output stability circle and input stability circle:





Transducer power gain:

Transducer power gain is nothing but the gain of the amplifier when placed between the source and load.

—

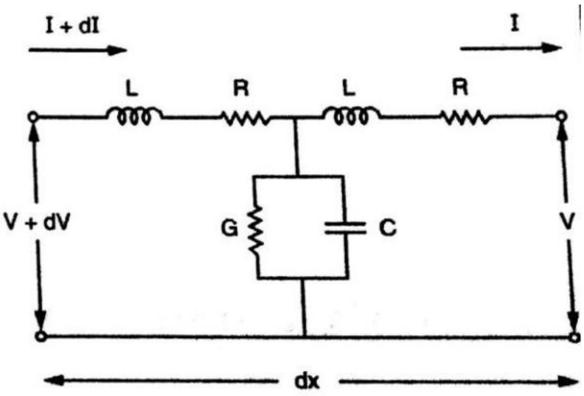
=

Relation between nodal quality factor (Q_n) with loaded quality factor (Q_L):

$Q_L = Q_n / 2$

UNIT – I - TRANSMISSION LINE THEORY

General Solution:



$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$$

$$V = A e^{\gamma x} + B e^{-\gamma x}$$

$$I = C e^{\gamma x} + D e^{-\gamma x}$$

$$I = A\sqrt{\frac{Y}{Z}}\, e^{\sqrt{ZY}\,x} - B\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{ZY}\,x} \qquad V = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{ZY}\,x} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{ZY}\,x}$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \qquad A = \frac{V_R}{2}\left[1 + \frac{Z_o}{Z_R}\right] \quad B = \frac{V_R}{2}\left[1 - \frac{Z_o}{Z_R}\right]$$

$$V_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \qquad C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right] \quad D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_o}\right]$$

$$V = \frac{V_R}{2}\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}\,x}$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_o}\right) e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_o}\right) e^{-\sqrt{ZY}\,x}$$

$$V = V_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + I_R Z_o\left(\frac{e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}}{2}\right) \quad [\because V_R = I_R Z_R]$$

$$I = I_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + \frac{V_R}{Z_o}\left(\frac{e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}}{2}\right) \quad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

$$V = V_R \cosh\sqrt{ZY}\ x + I_R Z_o \sinh\sqrt{ZY}\ x$$

$$I = I_R \cosh\sqrt{ZY}\ x + \frac{V_R}{Z_o} \sinh\sqrt{ZY}\ x$$

$$V_S = V_R\left[\cosh\sqrt{ZY}\ l + \frac{Z_0}{Z_R} \sinh\sqrt{ZY}\ l\right]$$

$$I_S = I_R\left[\cosh\sqrt{ZY}\ l + \frac{Z_R}{Z_0} \sinh\sqrt{ZY}\ l\right]$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$Z_S = \frac{Z_0 (Z_R \cosh\sqrt{ZY}\ l + Z_0 \sinh\sqrt{ZY}\ l)}{(Z_0 \cosh\sqrt{ZY}\ l + Z_R \sinh\sqrt{ZY}\ l)}$$

$$Z_S = Z_0\left[\frac{Z_R + Z_0 \tanh\gamma l}{Z_0 + Z_R \tanh\gamma l}\right]$$

$$Z_S = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}\right]$$

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}}\right]$$

Wavelength and velocity of propagation:

The propagation constant (γ) and characteristic impedance (Z_0) are called secondary constants of a transmission line.

Propagation constant is usually a complex quantity $\gamma = \alpha + j\beta$

$$\gamma = \sqrt{ZY}$$

The characteristic impedance of the transmission line is also a complex quantity.

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$\alpha + i\beta = \sqrt{RG - \omega^2 LC + j\omega(LG + RC)}$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC$$

$$2\alpha\beta = \omega(LG + RC)$$

$$\therefore \quad \beta = \sqrt{\frac{\omega^2LC - RG + \sqrt{(RG - \omega^2LC)^2 + \omega^2(LG + RC)^2}}{2}}$$

$$\therefore \quad \alpha = \sqrt{\frac{RG - \omega^2LC + \sqrt{(RG - \omega^2LC)^2 + \omega^2(LG + RC)^2}}{2}}$$

Velocity:

The velocity of propagation is given by,

$$v = \lambda f$$

$$= 2\pi f \frac{\lambda}{2\pi}$$

$$v = \frac{1}{\sqrt{LC}}$$

Wavelength:

The distance travelled by the wave along the line while the phase angle is changing through 2π radians is called wavelength.

$$\lambda = \frac{2\pi}{\beta} \quad \text{or} \quad \lambda = \frac{v}{f}$$

Waveform distortion:

The received waveform will not be identical with the input waveform at the sending end. This variation is known as distortion.

1. Frequency distortion
2. Delay or phase distortion

Frequency Distortion: A complex (voice) voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

Delay or Phase Distortion: For an applied voice-voltage wave the received waveform may not be identical with the input waveform at the sending end, since some frequency components will be delayed more than those of other frequencies. This phenomenon is known as delay or phase distortion.

The Distortion Less Line:

If a line is to have neither frequency nor delay distortion, then attenuation factor α and the velocity of propagation v cannot be functions of frequency.

$$\text{If} \qquad v = \frac{\omega}{\beta}$$

β must be a direct function of frequency

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

For β to be a direct function of frequency, the term $(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2$ must be equal to $(RG + \omega^2 LC)^2$

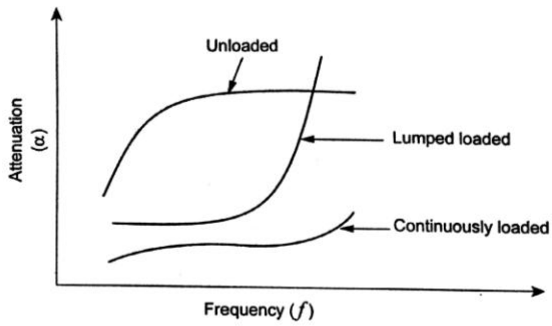
$$\frac{R}{L} = \frac{G}{C}$$

This is the condition for distortionless line.

Loading:

- To achieve distortion less condition $\rightarrow$ increase L/C ratio
- Increasing inductance by inserting inductances in series with the line is termed as loading such lines are called as loaded lines
- Lumped inductors $\rightarrow$ loading coils

(a) Lumped loading
(b) Continuous loading
(c) Patch loading



Comparison of loaded and unloaded cable characteristics

Inductance loading of Telephone cables:

Consider an uniformly loaded cable with G = 0. Then,

$$Z = R + j\omega L$$

$$Y = j\omega C$$

$$Z = \sqrt{R^2 + (L\omega)^2} \; \underline{\left| \tan^{-1}\left(\frac{L\omega}{R}\right) \right.}$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \omega\sqrt{LC}\ \sqrt[4]{1+\left(\frac{R}{L\omega}\right)^2}\ \angle\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

$$\therefore\ \gamma = \omega\sqrt{LC}\ \angle\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

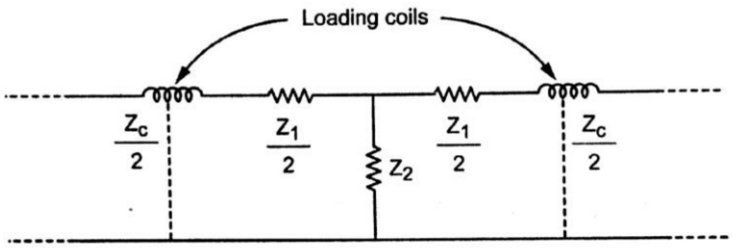
$$= \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\therefore \text{Attenuation constant } \alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\text{Phase-shift } \beta = \omega\sqrt{LC}$$

$$\text{Velocity of propagation } v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Campbell's Equation



Equivalent T section for part of a line between two lumped loading coils

$$\frac{Z_1'}{2} = \frac{Z_c}{2} + \frac{Z_1}{2}$$

$$\therefore\ \frac{Z_1'}{2} = \frac{Z_c}{2} + Z_o \tanh\frac{\gamma l}{2}$$

$$\cosh\gamma' l = \frac{Z_c}{2Z_o}\sinh\gamma l + \cosh\gamma l$$

Derive the expressions for open circuited and short circuited lines:

$$V_S = V_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_o}{Z_R}\sinh\sqrt{ZY}\,l\right]$$

$$I_S = I_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_o}\sinh\sqrt{ZY}\,l\right]$$

The input impedance of a transmission line is given by

$$Z_S = \frac{V_S}{I_S}$$

$$Z_S = Z_o \left(\frac{Z_R \cosh \gamma l + Z_o \sinh \gamma l}{Z_o \cosh \gamma l + Z_R \sinh \gamma l} \right)$$

Short circuited impedance

$$Z_{sc} = Z_o \tanh \gamma l$$

The open circuited impedance

$$Z_{oc} = Z_o \coth \gamma l$$

Reflection on a line not terminated in its characteristic impedance (Z_o):

When the load impedance is not equal to the characteristic impedance of a transmission line, reflection takes place, i.e., $Z_R \neq Z_0$, reflection occurs.

If a transmission line is not terminated in Z_0, then part of the wave is reflected back. The reflection is maximum when the line is open circuit or short circuit.

From the general solution of a transmission line, the equations for voltage and current are expressed as:

$$E = \frac{E_R(Z_R + Z_0)}{2Z_R}\left[e^{\gamma s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma s}\right]$$

$$I = \frac{I_R(Z_R + Z_0)}{2Z_R}\left[e^{\gamma s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma s}\right]$$

Incident voltage component is given by

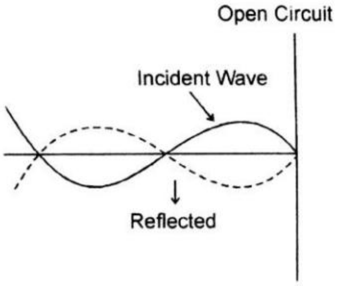
$$E_1 = \frac{E_R(Z_R + Z_0)}{2Z_R}e^{\gamma s} = \frac{E_R\left(1 + \frac{Z_0}{Z_R}\right)}{2}e^{\gamma s}$$

Reflected voltage component is given by,

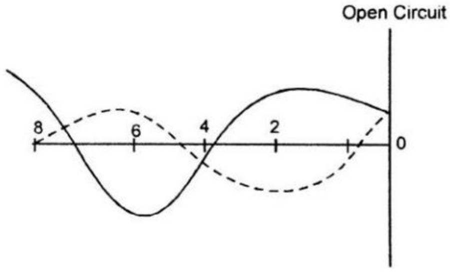
$$E_2 = \frac{E_R(Z_R - Z_0)}{2Z_R}e^{-\gamma s} = \frac{E_R\left(1 - \frac{Z_0}{Z_R}\right)}{2}e^{-\gamma s}$$

If $Z_R = \infty$ which represents an open circuited line,

At $s = 0$, both E_1 and E_2 have an amplitude of $E_R/2$. Thus at the receiving end, initial value of the reflected wave is equal to incident voltage.



(i) For time instant t = 0



(ii) For time instant t = 1/8 f

Input impedance and transfer impedance:

Input impedance :

The equations for voltage and current at the sending end of a transmission line of length 'l' are given by

$$V_S = V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)$$

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$Z_S = \frac{Z_0\,(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l)}{(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l)}$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the line is

$$Z_S = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

or $$Z_S = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right]$$

If $K = \frac{Z_R - Z_0}{Z_R + Z_0}$, then

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K\,e^{-\gamma l}}{e^{\gamma l} - K\,e^{-\gamma l}}\right]$$

Transfer impedance :

$$Z_T = \frac{V_S}{I_R}$$

$$Z_T = \frac{V_S}{I_R} = \frac{Z_R + Z_0}{2}\,(e^{\gamma l} + K\,e^{-\gamma l})$$

$$Z_T = Z_R\cosh\gamma l + Z_0\sinh\gamma l$$

Parameters of open-wire and coaxial lines:

The inductance of an open wire line is given by,

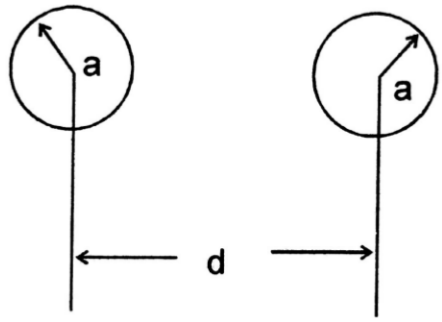
$$L = 10^{-7}\left(\frac{\mu}{\mu_{r}} + 4\ln\frac{d}{a}\right)$$

The first term on the right hand side of the above expression represents internal inductance of the line due to internal flux linkages in the conductors and is zero for a open wire line.

Hence the inductance of the open wire line is

$$L = 4 \times 10^{-7} \ln\frac{d}{a} \text{ henrys/m}$$

$$= 9.21 \times 10^{-7} \log_{10}\frac{d}{a} \text{ henrys/m}$$



$a \rightarrow$ radius of conductor

$d \rightarrow$ distance between conductors.

The value of capacitance of a line is not affected by skin effect or frequency and hence the capacitance of a open wire line with air dielectric is given by,

$$C = \frac{\pi\varepsilon_{v}\varepsilon_{r}}{\ln\frac{d}{a}} \text{ farads/m}$$

where ε_{v} = Permittivity of free space = 8.85 $\times$ 10^{-12} f/m,

$$\varepsilon_r = 1 \text{ for air}$$

$$C = \frac{27.7}{\ln\frac{d}{a}} \ \mu\mu\text{f/m}.$$

$$C = \frac{12.07}{\log_{10}\frac{d}{a}} \mu\mu f / m$$

The resistance of a round conductor of radius `a' meters to direct current is inversely proportional to the area as,

$$R_{dc} = \frac{k}{\pi a^2}$$

While that of a round conductor with alternating current flowing in a skin of thickness δ is,

$$R_{ac} = \frac{k}{2\pi a \delta}$$

Therefore the ratio of resistance to alternating current to resistance to direct current is given by,

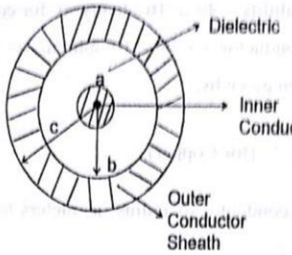
$$\frac{R_{ac}}{R_{dc}} = \frac{a\sqrt{\pi f \mu \sigma}}{2} = \frac{a}{2\delta}$$

For copper

$$\frac{R_{ac}}{R_{dc}} = 7.53 a\sqrt{f}$$

PARAMETERS OF THE COAXIAL LINE AT HIGH FREQUENCIES

Because of the skin effect, the current flows on the outer surface of the inner conductor and the inner surface of the outer conductor.



For a coaxial line the inductance is given by,

$$L = 10^{-7}\left[2\ln\frac{b}{a} + \frac{2C^4 \ln\frac{c}{b}}{(C^2 - b^2)^2} - \frac{C^2}{C^2 - b^2}\right] H/m$$

second term and third term represents flux linkages inside the inner and outer conductors.

The skin effect eliminates flux linkages and hence the inductance of coaxial line is given by,

$$L = 2 \times 10^{-7} \ln\frac{b}{a} \text{ henrys/m}$$

$$L = 4.6 \times 10^{-7} \log_{10}\frac{b}{a} \text{ henrys/m}$$

The capacitance of the coaxial line is not affected by the frequency.

$$C = \frac{2\pi\varepsilon}{\ln\frac{b}{a}} \text{ farads/m}$$

$$C = \frac{24.14\varepsilon_r}{\log_{10}\frac{b}{a}} \ \mu\mu\text{f/m.}$$

Due to skin effect resistance increases and the resistance of coaxial copper line is
Due to skin effect resistance increases and the resistance of coaxial copper line is

$$R_{ac} = 4.16 \times 10^{-8}\sqrt{f}\left[\frac{1}{b}+\frac{1}{a}\right]\Omega/m$$

The ac resistance of the coaxial cable is derived as follows,

$$R_{ac} = \frac{1}{2\pi a\delta\sigma}+\frac{1}{2\pi b\delta\sigma} = \frac{1}{2\pi\delta\sigma}\left[\frac{1}{a}+\frac{1}{b}\right]$$

The ac resistance per unit length of a copper conductor is given by,

$$R_{ac} = \frac{1}{2\pi\left(\frac{0.0664}{\sqrt{f}}\right)\left(5.75\times10^{7}\right)}\left[\frac{1}{a}+\frac{1}{b}\right]$$

$$R_{ac} = 4.16 \times 10^{-8}\sqrt{f}\left[\frac{1}{a}+\frac{1}{b}\right]\Omega/m.$$

The dc resistance of a coaxial line is given by,

$$R_{dc} = \frac{1}{\pi\sigma}\left[\frac{1}{a^2}+\frac{1}{(c^2-b^2)}\right] \ \Omega/\text{m}$$

1. Line constants for zero dissipation line:

In general the line constants for a transmission line are:

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

Characteristic impedance $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

Propagation constant $\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\gamma = \alpha + j\beta.$$

For a transmission of energy at high frequencies, $\omega L >> R$. We assume negligible losses or zero dissipation and G is also assumed to be zero. $(G = 0)$

Using the inductance and capacitance a **open wire line at high frequency**, the value of characteristic impedance of the open wire line can be found as,

$$L = 4 \times 10^{-7} \ln \frac{d}{a} \text{ h/m} \qquad C = \frac{27.7}{\ln d/a} \ \mu\mu\text{f/m}$$

$$R_0 = \sqrt{\frac{L}{C}} = 120 \ln \frac{d}{a} \text{ ohms.}$$

(or) $$R_0 = 276 \log_{10} d \text{ ohms.}$$

The characteristic impedance of the coaxial line can be computed as,

$$L = 4.60 \times 10^{-7} \log_{10} b/a \text{ h/m}$$

$$C = \frac{24.14\varepsilon_r}{\log_{10} b/a} \ \mu\mu\text{F/m}$$

$$R_0 = \sqrt{\frac{L}{C}} = \frac{138}{\sqrt{\varepsilon_r}} \log_{10} b/a \text{ ohms}$$

$$L = 2 \times 10^{-7} \ln b/a \text{ h/m}$$

$$C = \frac{55.5\varepsilon_r}{\ln b/a} \ \mu\mu\text{f/m}$$

$$R_0 = \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\varepsilon_r}} \ln \frac{b}{a} \text{ ohms}$$

→ **The propagation contant** g is given by,

$$\gamma = \sqrt{ZY} = \sqrt{(-j\omega L)(j\omega C)} = \sqrt{j^2\omega^2 LC} = j\omega\sqrt{LC}$$

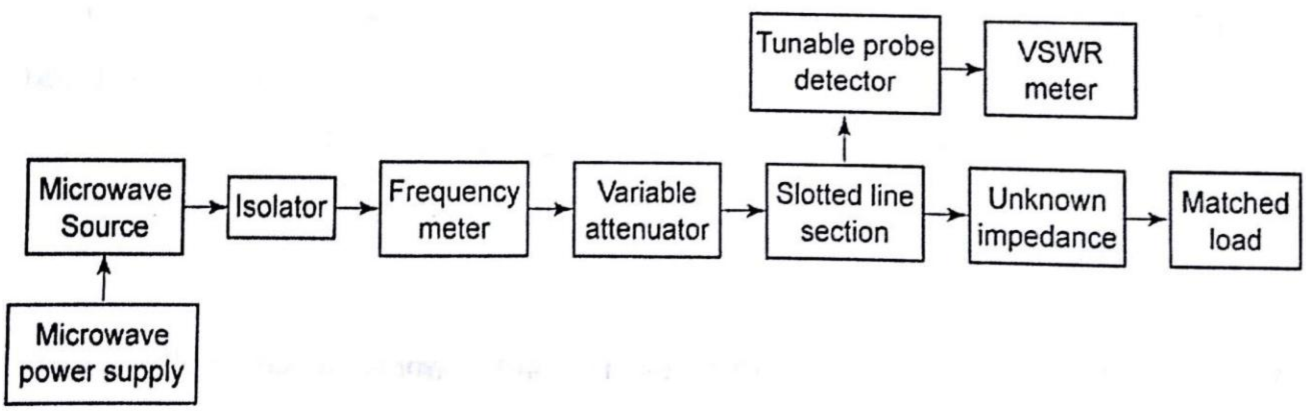
$$= \sqrt{-j\omega^2 LC}$$

The velocity of propagation can be calculated as

$$V = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \text{ m/sec.}$$

Measurement of VSWR and Wavelength:

- ***VSWR*** and the ***magnitude of voltage reflection*** coefficient are very important parameters which determine the ***degree of impedance matching***.
- VSWR and Γ are also used for measurement of ***load impedance*** by the slotted line method.



- When a load $Z_L \neq Z_o$ is connected to the transmission line, the standing waves are produced.
- By inserting a slotted line system in the line, ***standing waves*** can be traced by moving the carriage with a tunable probe detector along the line.
- VSWR can be measured by detecting V_{max} and V_{min} in the ***VSWR meter.***

$$\text{Standing wave ratio (S)} = \frac{V_{max}}{V_{min}} = \frac{1+\Gamma}{1-\Gamma} \qquad \ldots (1)$$

$$\Gamma = \text{Reflection coefficient} = \frac{P_{reflected}}{P_{incident}} \qquad \ldots (2)$$

- Here, $P_{reflected}$ is a reflected power and $P_{incident}$ is a incident power of unknown impedance. S varies from ***1 to ∞***.As Γ varies from ***0 to ∞***.

LOW VSWR (S < 20)

- Values of VSWR ***not exceeding 20*** are very easily measured ***directly on the VSWR meter*** using the experimental set-up shown in Fig.18.9 as follows,
  (1) The variable attenuator is adjusted to ***10dB.*** The microwave source is set to the required frequency. The ***1kHz modulation is adjusted*** for maximum reading on the VSWR meter in a 30dB scale.
  (2) The probe on the slotted waveguide is moved to get ***maximum*** reading on the meter (corresponding to V_{max}).
  (3) The attenuation is now adjusted to get full-scale reading. This full-scale reading is noted down. Next the probe on the slotted line is adjusted to get ***minimum*** reading on the meter (corresponding to V_{min}).
  (4) The ratio of $\frac{V_{max}}{V_{min}}$ gives the ***VSWR.***
- The experiment is repeated for other frequencies as required to obtain a set of values of *S* Vs *f*.

The Possible Sources of Error in this Measurements are:

(1) V_{max} and V_{min} ***may not be*** measured in the ***square – law region*** of the ***crystal detector.***

(2) The probe thickness and depth of the penetration may produce ***reflections*** in the line and also ***distortion*** in the field to be measured.

(3) When ***VSWR < 1.05,*** the associated VSWR of connector produces ***significant error*** in VSWR measurement. Very good ***low VSWR (<1.01)*** connectors should be used for very low VSWR measurements.

- For high power, ***double minimum method is used.*** The electromagnetic field at any point of transmission line may be considered as the sum of two traveling waves: the ***'Incident wave'*** which propagates from generator and ***'reflected wave'*** which propagates towards the generator.
- The reflected wave is set up by reflection of incident wave from a discontinuity on the line or from the load impedance.
- The magnitude and phase of reflected wave depends upon amplitude and phase of the reflecting impedance.
- The superposition of two traveling waves, gives rise to standing wave along the line.
- The maximum field strength is found where two waves are in phase and it is minimum where two waves adds in an opposite phase.
- The distance between two successive minimums (or maximums) is half the guide wavelength on the line.

Reflection Coefficient:

The ratio of electrical field strength of reflected and incident wave is called the reflection coefficient.

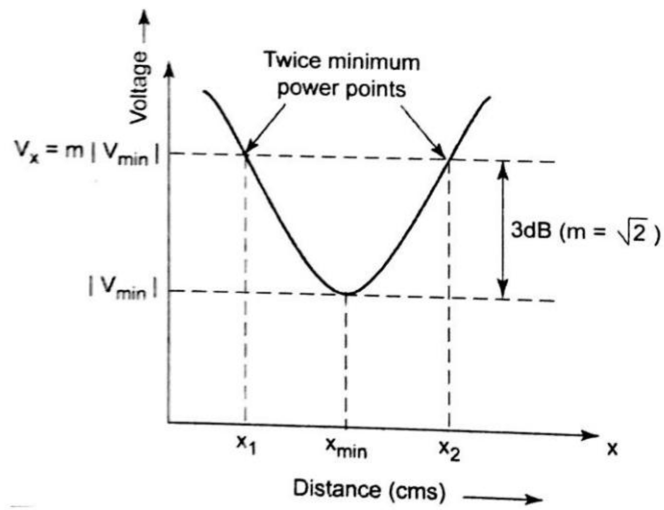
- Reflection coefficient, ρ is

$$\Gamma = \frac{E_r}{E_I} = \frac{Z - Z_0}{Z + Z_0}$$

where, Z is the impedance at a point, and

Z_0 is characteristic impedance.

- The above equation gives following equation $|\Gamma| = \frac{S-1}{S+1}$



Double minima method

VSWR:

- VSWR denoted by S is,

$$S = \frac{E_{max}}{E_{min}} = \frac{|E_I| + |E_r|}{|E_I| - |E_r|}$$

where,

E_I – Incident voltage, and

E_r – Reflected voltage.

- In this method, the probe is inserted to a depth where the minimum can be real without difficulty.
- The probe is then moved to a point where the power is ***twice the minimum.*** Let this position be denoted by x_1.
- The probe is then moved to ***twice the power*** point on the other side of the minimum (say x_2).

$$P_{min} \propto V_{min}^2$$

$$2\,P_{min} \propto V_x^2$$

$$\frac{1}{2} = \frac{V_{min}^2}{V_x^2}$$

$$V_x^{\,2} = 2\left(V_{min}\right)^2$$

$$V_x = \sqrt{2}\,V_{min}$$

Guide Wavelength:

- By moving the probe between two successive minima, a distance equal to $\frac{\lambda_g}{2}$ is found to determine the guide wavelength λ_g.

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_C}\right)^2}}$$

Quarter wave line and Half wave line:

The input impedance of a dissipationless transmission line is

$$Z_S = R_0\left[\frac{Z_R + j\,R_0 \tan\beta x}{R_0 + j\,Z_R \tan\beta x}\right]$$

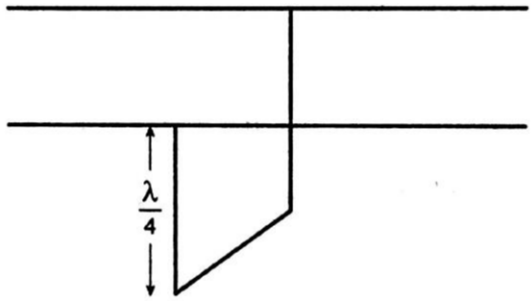
$$Z_S = R_0\left[\frac{\frac{Z_R}{\tan\beta x} + j\,R_0}{\frac{R_0}{\tan\beta x} + j\,Z_R}\right]$$

For a quarter wave line $x = \lambda/4$,

$$\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_S = R_0 \left[\frac{\dfrac{Z_R}{\tan \pi/2} + j\, R_0}{\dfrac{R_0}{\tan \pi/2} + j\, Z_R} \right] = R_0 \left[\frac{j\, R_0}{j\, Z_R} \right]$$

$$Z_S = \frac{R_0^2}{Z_R}$$



Half-Wave Line

The input impedance of a dissipationless transmission line is

$$Z_S = R_0 \left[\frac{Z_R + j\, R_0 \tan \beta x}{R_0 + j\, Z_R \tan \beta x} \right]$$

For a half-wave line $x = \lambda/2$

$$\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$Z_S = R_0 \left[\frac{Z_R + j\, R_0 \tan \pi}{R_0 + j\, Z_R \tan \pi} \right]$$

$$= R_0 \frac{Z_R}{R_0}$$

$$Z_S = Z_R$$

Single stub matching:

Location and length of the stub using reflection coefficient:

The input impedance of the line is given by

$$Z_i = Z_0 \frac{1 + K e^{-2\gamma l}}{1 - K e^{-2\gamma l}}$$

For lossless line $\alpha = 0$, $\gamma = j\beta$ and $K = |K| e^{j\phi}$

where ϕ is the angle of reflection coefficient.

$$Z_i = Z_0 \frac{1 + |K| e^{j\phi} e^{-j2\beta l}}{1 - |K| e^{j(\phi - 2\beta l)}}$$

$$= Z_0 \frac{1 + |K| e^{j(\phi - 2\beta l)}}{1 - |K| e^{j\phi} e^{-j2\beta l}}$$

The input admittance is given by

$$Y_i = G_0 \frac{1 - |K| e^{j(\phi - 2\beta l)}}{1 + |K| e^{j(\phi - 2\beta l)}}$$

where the characteristic conductance is

$$G_0 = \frac{1}{Z_0} = \frac{1}{R_0} \qquad [\because Z_0 \text{ is resistive}]$$

$$Y_i = G_0 \frac{1 - |K| [\cos(\phi - 2\beta l) + j \sin(\phi - 2\beta l)]}{1 + |K| [\cos(\phi - 2\beta l) + j \sin(\phi - 2\beta l)]}$$

$$= G_0 \frac{1 - |K| [\cos(\phi - 2\beta l) - j |K| \sin(\phi - 2\beta l)]}{1 + |K| [\cos(\phi - 2\beta l) + j |K| \sin(\phi - 2\beta l)]}$$

Multiplying the numerator and denominator by

$1 + |K| [\cos(\phi - 2\beta l) - j |K| \sin(\phi - 2\beta l)$

$$Y_i = G_0 \frac{1 - |K|^2 - 2j |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Since $Y_i = G_i + j S_i$, then

$$\frac{Y_i}{G_0} = \frac{G_i}{G_0} + \frac{j S_i}{G_0} = \frac{1 - |K|^2 - 2j |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Equating the real parts

$$\frac{G_i}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Equating the imaginary parts

$$\frac{S_i}{G_0} = \frac{-2|K|\sin(\phi - 2\beta l)}{1 + |K|^2 + 2|K|\cos(\phi - 2\beta l)}$$

At the location of stub $Z_i = Z_0$ for matching.

Since there is no reflection, $l = l_s$

$$\therefore G_i = G_0$$

$$\frac{G_i}{G_0} = 1$$

$$\frac{1 - |K|^2}{1 + |K|^2 + 2|K|\cos(\phi - 2\beta l_s)} = 1$$

$$1 - |K|^2 = 1 + |K|^2 + 2|K|\cos(\phi - 2\beta l_s)$$

$$2|K|\cos(\phi - 2\beta l_s) = -2|K|^2$$

$$\cos(\phi - 2\beta l_s) = -|K|$$

$$\phi - 2\beta l_s = \cos^{-1}(-|K|)$$

$$\text{But} \quad \cos^{-1}(-|K|) = -\pi + \cos^{-1}|K|$$

$$\therefore \quad \phi - 2\beta l_s = -\pi + \cos^{-1}|K|$$

$$2\beta l_s = \phi + \pi - \cos^{-1}|K|$$

$$l_s = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}$$

$$\text{or} \quad l_s = \frac{\lambda}{4\pi}[\phi + \pi - \cos^{-1}|K|] \qquad [\because \beta = \frac{2\pi}{\lambda}]$$

The normalized susceptance (imaginary part) equation is

$$\frac{S_i}{G_0} = \frac{-2|K|\sin(\phi - 2\beta l)}{1 + |K|^2 + 2|K|\cos(\phi - 2\beta l)}$$

$$\text{But} \quad (\phi - 2\beta l_s) = -\pi + \cos^{-1}|K| \quad \text{and}$$

$$\cos(\phi - 2\beta l_s) = -|K|$$

$$\therefore \quad \frac{S_i}{G_0} = \frac{-2|K|\sin(-\pi + \cos^{-1}|K|)}{1 + |K|^2 + 2|K|(-|K|)}$$

$$= \frac{2|K|\sin(\cos^{-1}|K|)}{1 + |K|^2 - 2|K|^2}$$

Let $\cos^{-1}|K| = \theta$, then $|K| = \cos\theta$ and

$$\sin(\cos^{-1}|K|) = \sin\theta$$

$$= \sqrt{1 - \cos^2\theta} = \sqrt{1 - |K|^2}$$

$$\therefore \frac{S_i}{G_0} = \frac{2|K|\sqrt{1 - |K|^2}}{1 - |K|^2}$$

$$S_i = G_0 \frac{2|K|}{\sqrt{1 - |K|^2}}$$

The susceptance of the stub is $G_0 \cot\beta l_t$

$$G_0 \cot\beta l_t = G_0 \frac{2|K|}{\sqrt{1 - |K|^2}}$$

$$\frac{1}{\tan \beta l_t} = \frac{2|K|}{\sqrt{1-|K|^2}}$$

$$\tan \beta l_t = \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$\beta l_t = \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$l_t = \frac{1}{\beta} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

The location of the stub 'l_s' and length of the stub 'l_t' can be determined, if the reflection coefficient and frequency are known.

Determine the stub length and the distance of the stub from the load. Given that a complex load ZL = 50 – j100 ohms is to be matched to a 75 ohms transmission line using a short circuited stub.

(a) Characteristic impedance (Z_0) of the line = 75Ω

Load impedance $Z_L = 50 - j100\ \Omega$

Normalized load impedance = $Z'_L = \frac{Z_L}{Z_0} = \frac{50 - j100\Omega}{75\Omega}$

$$Z'_L = 0.667 - j1.33$$

Normalized load impedance Z_L' is plotted at the intersection of constant R circle with $R =$ 0.67 and with $X = 1.33$. This is point A. The impedance circle is drawn.

(b) The normalized load admittance point B is determined by drawing a line from point A through the center to the opposite side of the S circle. (i.e., Point B),

$$Y = 0.3 + j0.6\ \mho.$$

(c) Travel along the constant S circle in the clockwise direction from load to generator to reach a point C on $g_t = 1$ circle (or) $\frac{Y}{G_0} = 1$ circle. Draw a line from O to C and extend the line to C' on the outer rim.

(d) The distance between $B'C'$ gives the distance of the stub from the load.

i.e., $s = 0.18\ \lambda - 0.09\ \lambda = 0.09\ \lambda$.

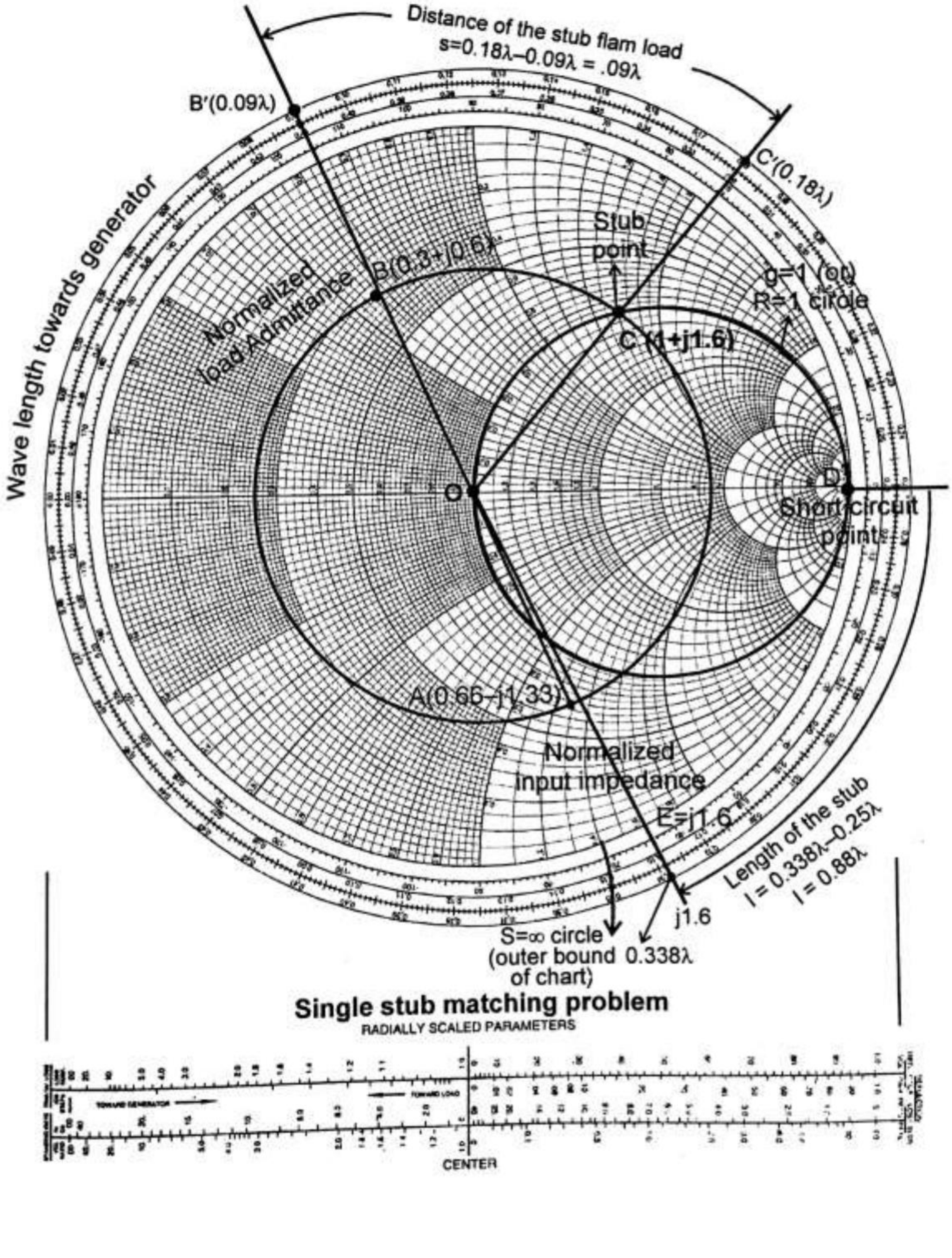
(e) At the point C, the normalized admittance value is $1 + j1.6$. This is the point at which the stub is connected. Thus the stub should provide a susceptance of – j1.6.

(f) The determine the length of the shorted stub that has opposite reactive component to the input admittance, the outside of the smith chart ($g = 0$ circle) is moved around until a point with susceptance of $-j1.6$ is reached which is point E. The point E represents a susceptance of –j1.6.

(g) The distance between D and E is the length of the sub length of the stub l,

$$= 0.338\ \lambda - 0.25\ \lambda$$

$$l = .088\ \lambda.$$



Single stub matching problem

A 50 Ω loss less feeder line is to be matched for an antenna with Z_L = (75-j20) Ω at 100MHz using single shorted stub. Calculate the stub length and distance between the antenna and the stub using smith chart.

Given:

$$Z_0 = 50\ \Omega$$

$$Z_L = (75 - j20)\ \Omega$$

$$f = 100\text{ MHz}$$

$$= \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6}$$

$$= 3\text{ m}$$

Normalized load impedance $\bar{z}_L = \frac{Z_L}{Z_0} = \frac{75 - j20}{50} = 1.5 - j0.4$

Normalized load impedance $\bar{z}_L$ is plotted at P on the Smith chart and the impedance circle with 'O' centre and OP as radius is drawn.

From the smith chart OS read as SWR = 1.7

The normalized load admittance is diametrically opposite to the normalized load impedance at Q *i.e.,* $\bar{y}_L = 0.62 + j0.17$.

$\bar{y}_L$ is moved in clockwise direction to a point A on the impedance circle where it intersects R = 1 circle *i.e.,* at $1 + j0.525$.

The distance between Q and A is the distance from the load to the location of the stub.

$$d = 0.1455\,\lambda - 0.0415\,\lambda$$

$$= 0.104\,\lambda$$

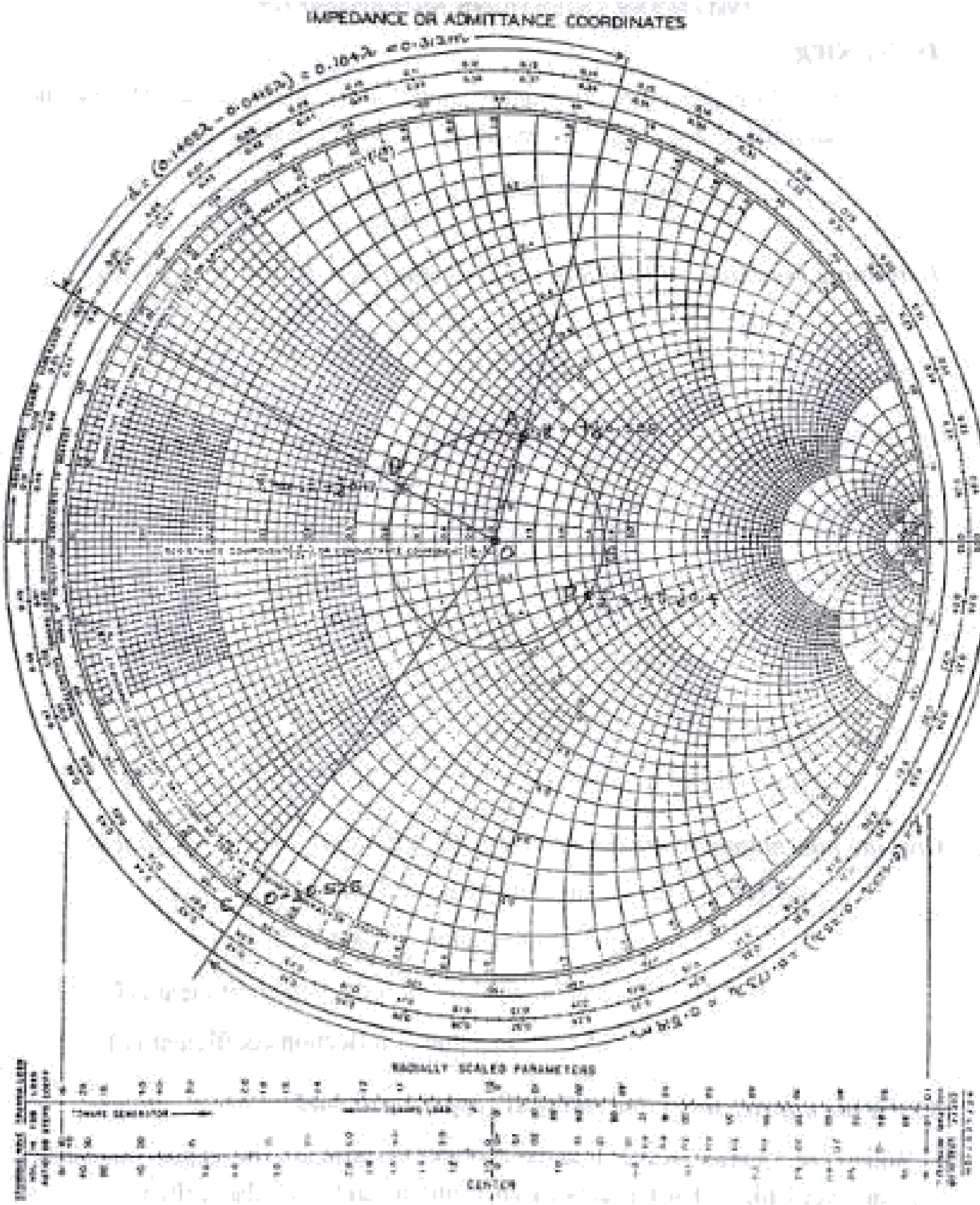
$$= 0.104 \times 3 = 0.312\text{ m}$$

The stub must have zero resistance and susceptance that has an exactly opposite value at C *i.e.,* $y_{stub} = 0 - j0.525$.

The length of the stub is measured from the right side of the chart (X = 0) at B to the point C.

$$l = (0.423 - 0.25)\,\lambda = 0.173\,\lambda$$

$$= 0.173 \times 3 = 0.519\text{ m}$$



A 75 Ω lossless line transmission line is to be match with a 100-j80Ω load using single stub. Calculate the stub length and its distance from the load corresponding to the frequency of 30 MHz using Smith Chart:

Given:

$$Z_0 = 75\,\Omega$$

$$Z_L = 100 - j80\,\Omega$$

$$f = 30\text{ MHz}$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^8}{30 \times 10^6}$$

$$= 10\text{ m}$$

Normalized load impedance $z_L = \frac{Z_L}{Z_0}$

$$= \frac{100 - j80}{75} = 1.33 - j1.067$$

The normalized load impedance z_L is marked in Smith chart at P and the impedance circle with $(1 + j0)$ as centre 'O' and 'OP' as radius.

$$\text{From chart SWR} = \text{OS} = 2.5$$

The normalized load admittance is diametrically opposite to the normalized load impedance at Q is $\bar{y}_L = 0.46 + j0.36$. $\bar{y}_L$ is moved in clockwise direction to a point A on the impedance circle where it intersects R = 1 circle *i.e.,* at $1 + j0.956$.

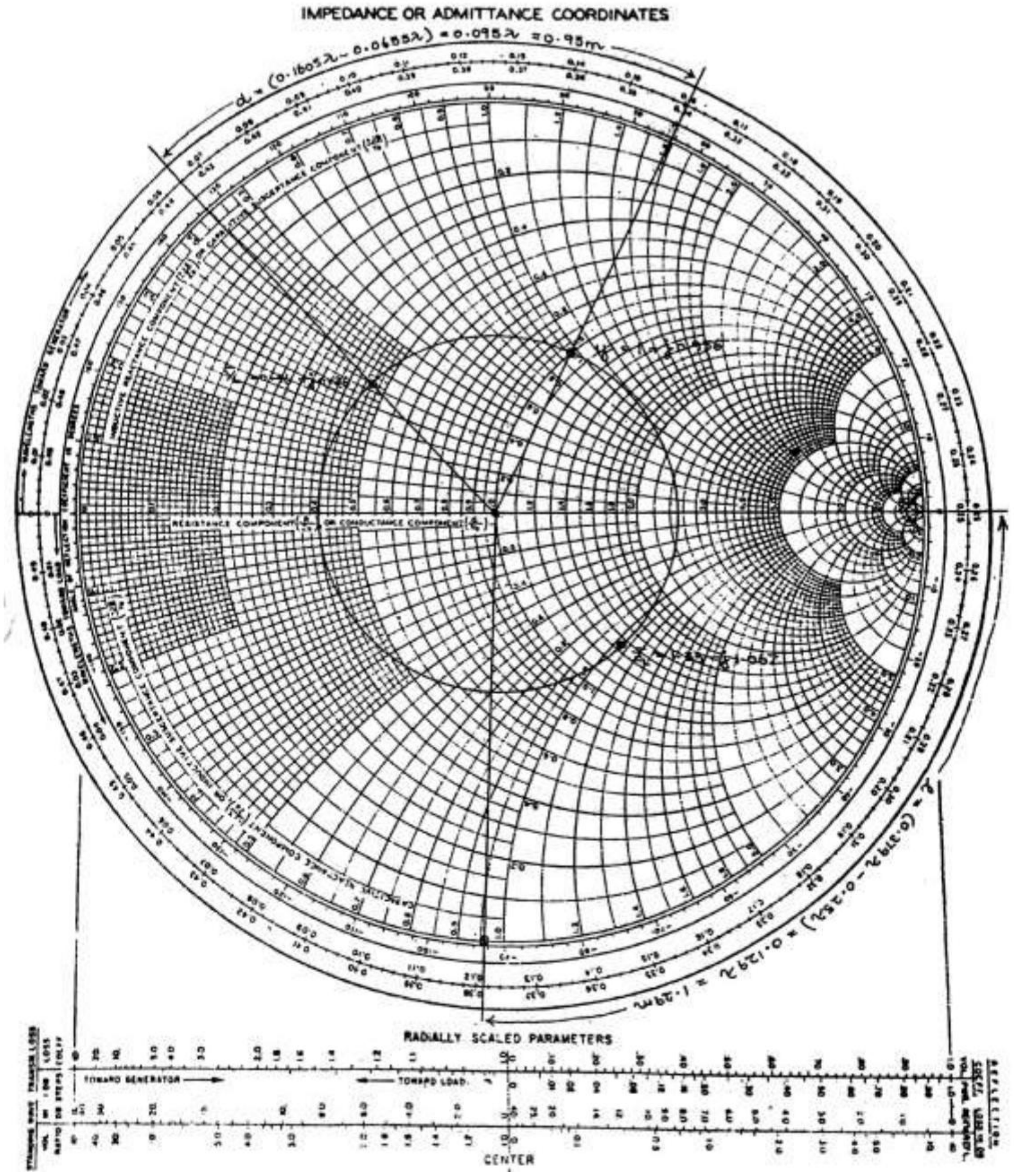
The distance between Q and A is the distance from the load to the location of the stub

$$d = 0.1605\,\lambda - 0.0655\,\lambda = 0.095\,\lambda$$
$$= 0.095 \times 10 = 0.95 \text{ m}$$

The stub must have zero resistance and susceptance that has an exactly opposite value at C *i.e.,* $y_{stub} = 0 - j0.095$.

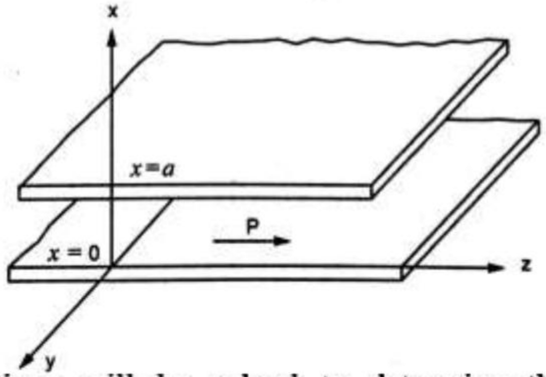
The length of the stub is measured from the right side of the chart (X = 0) at B to the point C.

$$l = (0.379 - 0.25)\,\lambda = 0.129\,\lambda$$
$$= 0.129 \times 10 = 1.29 \text{ m}$$



Electric field and magnetic field expression between the parallel plates:

Consider an electromagnetic wave propagating between a pair of parallel perfectly conducting planes of infinite extent in the y and z directions as shown in Fig



Maxwell's equations will be solved to determine the electromagnetic field configurations in the rectangular region.

Maxwell's equations for a non-conducting rectangular region are given as

$$\nabla \times H = j\,\omega\,\varepsilon\,E$$
$$\nabla \times E = -j\,\omega\,\mu\,H$$

$$\nabla \times H = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \bar{a}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \bar{a}_y\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \bar{a}_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= j\,\omega\,\varepsilon\,[\bar{a}_x E_x + \bar{a}_y E_y + \bar{a}_z E_z]$$

Equating x, y and z components on both sides,

$$\left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z \end{aligned}\right\}$$

Similarly, $$\nabla \times E = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \bar{a}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \bar{a}_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \bar{a}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= -j\,\omega\,\mu\,[\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z\, H_z]$$

Equating x, y and z components on both sides,

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega\mu H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega\mu H_z \end{aligned}\right\}$$

The wave equation is given by

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

where $$\gamma^2 = (\sigma + j\omega\varepsilon)\;(j\omega\mu)$$

For a non-conducting medium, it becomes

$$\nabla^2 E = -\,\omega^2\mu\varepsilon E$$

$$\nabla^2 H = -\,\omega^2\mu\varepsilon H$$

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} &= -\,\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} &= -\,\omega^2\mu\varepsilon H \end{aligned}\right\}$$

It is assumed that the propagation is in the z direction and the variation of field components in this z direction may be expressed in the form $e^{-\gamma z}$,

where γ is propagation constant.

$$\gamma = \alpha + j\beta$$

If $\alpha = 0$, wave propagates without attenuation.

If α is real *i.e.*, $\beta = 0$, there is no wave motion but only an exponential decrease in amplitude.

Let $$H_y = H_y^0\; e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma\, H_y^0\; e^{-\gamma z} = -\gamma\, H_y$$

Similarly, $$\frac{\partial H_x}{\partial z} = -\gamma\, H_x$$

Let
$$E_y = E_y^0\, e^{-\gamma z}$$
$$\frac{\partial E_y}{\partial z} = -\gamma\, E_y$$
Similarly
$$\frac{\partial E_x}{\partial z} = -\gamma\, E_x$$
There is no variation in the *y* direction *i.e.*, derivative of *y* is zero
$$\left.\begin{aligned} \gamma H_y &= j\omega\varepsilon E_x \\ -\gamma H_x - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} &= j\omega\varepsilon E_z \end{aligned}\right\}$$
$$\left.\begin{aligned} \gamma E_y &= -j\omega\mu H_x \\ -\gamma E_x - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} &= -j\omega\mu H_z \end{aligned}\right\}$$
$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \gamma^2 E &= -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \gamma^2 H &= -\omega^2\mu\varepsilon H \end{aligned}\right\}$$
$$\text{where} \quad \frac{\partial^2 E}{\partial z^2} = \gamma^2 E \text{ and } \frac{\partial^2 H}{\partial z^2} = \gamma^2 H$$
To solve H_x,
$$-\gamma\, H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$
$$\gamma E_y = -j\omega\mu H_x$$
From the above equations,
$$H_x = \frac{-\gamma\, E_y}{j\omega\mu}$$
$$E_y = -\frac{1}{j\omega\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]$$
Substituting the value of E_y in the above equation,
$$H_x = \frac{-\gamma}{j\omega\mu}\left[-\frac{1}{j\omega\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)\right]$$
$$H_x = \frac{-\gamma}{\omega^2\mu\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]$$
$$H_x\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right] = \frac{-\gamma}{\omega^2\mu\varepsilon}\left[\frac{\partial H_z}{\partial x}\right]$$
$$H_x\,[\,\omega^2\mu\varepsilon + \gamma^2\,] = -\gamma\,\frac{\partial H_z}{\partial x}$$

$$\boxed{H_x = \frac{-\gamma}{\omega^2\mu\varepsilon + \gamma^2}\frac{\partial H_z}{\partial x}}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$\text{where } \quad h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

To solve H_y,

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\gamma H_y = j\omega\varepsilon E_x$$

From the above equations,

$$H_y = \frac{j\omega\varepsilon}{\gamma} E_x$$

$$E_x = \frac{1}{\gamma}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

Substituting the value of E_x in the above equation,

$$H_y = \frac{j\omega\varepsilon}{\gamma}\cdot\frac{1}{\gamma}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

$$H_y = \frac{-\omega^2\mu\varepsilon}{\gamma^2} H_y - \frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y\left(1 + \frac{\omega^2\mu\varepsilon}{\gamma^2}\right) = -\frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{(\gamma^2 + \omega^2\mu\varepsilon)}\frac{\partial E_z}{\partial x}$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\boxed{H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

To solve E_x,

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$H_y = \frac{j\omega\varepsilon}{\gamma} E_x$$

Substituting the value of H_y in the above equation,

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\left[\frac{j\omega\varepsilon}{\gamma} E_x\right]$$

$$= \frac{-\omega^2\mu\varepsilon}{\gamma} E_x$$

$$\gamma E_x + \frac{\omega^2\mu\varepsilon}{\gamma} E_x = -\frac{\partial E_z}{\partial x}$$

$$E_x\left[\gamma + \frac{\omega^2\mu\varepsilon}{\gamma}\right] = -\frac{\partial E_z}{\partial x}$$

$$E_x[\gamma^2 + \omega^2\mu\varepsilon] = -\gamma \frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x}$$

To solve E_y,

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y$$

$$H_x = -\frac{\gamma E_y}{j\omega\mu}$$

Substituting the value of H_x in the above equation,

$$\frac{-\gamma^2 E_y}{j\omega\mu} + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y$$

$$E_y\left[\frac{-\gamma^2}{j\omega\mu} + j\omega\varepsilon\right] = -\frac{\partial H_z}{\partial x}$$

$$E_y[\gamma^2 + \omega^2\mu\varepsilon] = j\omega\mu \frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

where $$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

Electric field and magnetic field expressions for TE waves between parallel plates:

Transverse electric (TE) waves are waves in which the electric field strength E is entirely transverse. It has a magnetic field strength H_z in the direction of propagation and no component of electric field E_z in the same direction. ($E_z = 0$).

Substituting the value of $E_z = 0$ in the following equations.

$$E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} \quad \text{and} \quad H_y = \frac{-j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}$$

Then $E_x = 0$ and $H_y = 0$

The wave equation for the component E_y

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \mu\varepsilon E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\omega^2\mu\varepsilon E_y - \gamma^2 E_y$$

But $$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0}$$

This is a differential equation of simple harmonic motion. The solution of this equation is given by

$$E_y = C_1 \sin hx + C_2 \cos hx$$

where C_1 and C_2 are arbitrary constants.

If E_y is expressed in time and direction ($E_y = E_y^0 \, e^{-\gamma z}$), then the solution becomes,

$$E_y = (C_1 \sin hx + C_2 \cos hx)\, e^{-\gamma z}$$

The arbitrary constants C_1 and C_2 are determined from the boundary conditions.

The tangential component of E is zero at the surface of conductors for all values of z.

$$E_y = 0 \text{ at } x = 0$$

$$E_y = 0 \text{ at } x = a$$

Applying the first boundary condition ($x = 0$)

$$0 = 0 + C_2$$

$$C_2 = 0$$

Then $$E_y = C_1 \sin hx \; e^{-\gamma z}$$

Applying the second boundary condition ($x = a$)

$$\sin ha = 0$$

$$h = \frac{m\pi}{a}$$

where $$m = 1, 2, 3, \ldots\ldots\ldots$$

Therefore, $$E_y = C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

$\therefore$ $$\frac{\partial E_y}{\partial x} = \frac{m\pi}{a} C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

$$\gamma E_y = -j\omega\mu H_x$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

From the first equation, $$H_x = \frac{-\gamma E_y}{j\omega\mu}$$

Substituting the value of E_y in the above equation

$$H_x = \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma x}$$

From the second equation, $$H_z = -\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial x}$$

Substituting the value of E_y in the above equation

$$H_z = \frac{-m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

$$H_z = \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

The field strengths for TE waves between parallel planes are

$$\left.\begin{aligned} E_y &= C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-\gamma z} \\ H_x &= \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-\gamma z} \\ H_z &= \frac{-m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right)e^{-\gamma z} \end{aligned}\right\}$$

$$\gamma = j\beta$$

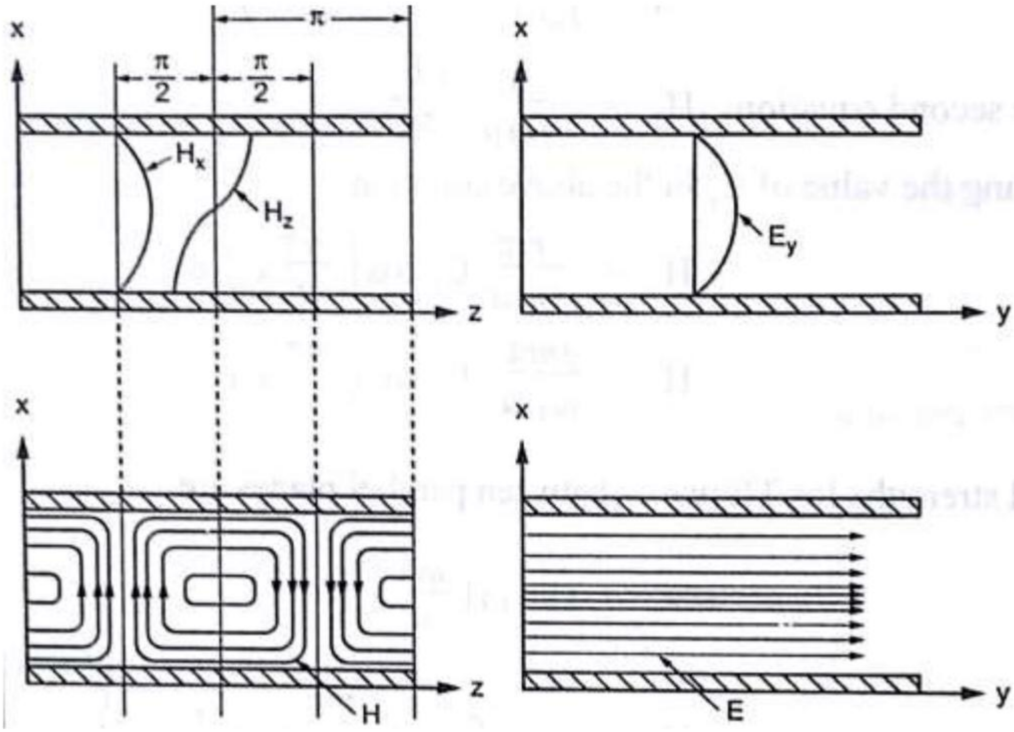
Then the field strengths for TE waves are

$$E_y = C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-j\beta z}$$

$$H_x = \frac{-\beta}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-j\beta z}$$

$$H_z = \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right)e^{-j\beta z}$$

The field distributions for TE_{10} mode between parallel planes are shown in Fig.



Electric field and magnetic field expressions for TE waves between rectangular waveguides:

The wave equation in a rectangular waveguide is given by

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu\varepsilon H_z$$

The solution of the equation is

$$H_z(x, y, z) = H_z^{\circ}(x, y)\, e^{-\gamma z}$$

Let $$H_z^{\circ}(x, y) = XY$$

where X is the function of x only.

Y is the function of y only.

Substituting the value of H_z in the wave equation,

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + \gamma^2 XY = -\omega^2 \mu\varepsilon XY$$

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + h^2 XY = 0$$

where $h^2 = \gamma^2 + \omega^2\mu\varepsilon$.

Dividing by XY,

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = \frac{-1}{Y}\frac{d^2Y}{dy^2}$$

The expression relates a function of x alone to a function of y alone and this can be equated to a constant.

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = A^2$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 - A^2 = 0$$

Let $$B^2 = h^2 - A^2$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + B^2 = 0$$

The solution of this equation is

$$X = C_1 \cos Bx + C_2 \sin Bx$$

Similarly, $$-\frac{1}{Y}\frac{d^2Y}{dy^2} = A^2$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} + A^2 = 0$$

The solution of this equation is $Y = C_3 \cos Ay + C_4 \sin Ay$

But $H_z^{\circ} = XY$

$$= (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

$$= C_1 C_3 \cos Ay \cos Bx + C_2 C_3 \cos Ay \sin Bx + C_1 C_4 \cos Bx \sin Ay + C_2 C_4 \sin Ay \sin Bx$$

It is known that

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

For TE waves $E_z = 0$.

$$E_x = -\frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

$$= -\frac{j\omega\mu}{h^2} [-C_1 C_3 A \sin Ay \cos Bx - C_2 C_3 A \sin Ay \sin Bx + C_1 C_4 A \cos Bx \cos Ay + C_2 C_4 A \cos Ay \sin Bx]$$

Applying boundary conditions, $E_z = 0$ when $y = 0$, $y = b$.

If $y = 0$, the general solution is

$$E_x = -\frac{j\omega\mu}{h^2} [C_1 C_4 A \cos Bx + C_2 C_4 A \sin Bx] = 0$$

For $E_x = 0$, $C_4 = 0$. (C_4 is common)

Then the general solution is

$$E_x = \frac{-j\omega\mu}{h^2} [-C_1 C_3 A \sin Ay \sin Bx - C_2 C_3 A \sin Ay \sin Bx]$$

If $y = b$, $E_x = 0$.

For $E_x = 0$, it is possible either $B = 0$ or $A = \frac{n\pi}{b}$. If $B = 0$, the above solution is lentically zero. So it is better to select $A = \frac{n\pi}{b}$.

The general solution is

$$E_x^{\circ} = \frac{j\omega\mu}{h^2} [C_1 C_3 A \sin Ay \cos Bx + C_2 C_3 A \sin Ay \sin Bx]$$

Similarly for E_y,

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$= \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} \qquad [\because E_z = 0]$$

$$= \frac{j\omega\mu}{h^2} [-C_1 C_3 B \cos Ay \sin Bx + C_2 C_3 B \cos Ay \cos Bx - C_1 C_4 B \sin Bx \sin Ay + C_2 C_4 B \sin Ay \cos Bx]$$

Applying boundary conditions

$$E_y = 0; \quad x = 0 \text{ and } x = a$$

If $x = 0$,

$$E_y^\circ = \frac{j\omega\mu}{h^2}\,[\,C_2\,C_3\,B\cos Ay + C_2\,C_4\,B\sin Ay\,]$$

For $E_y^\circ = 0,\ C_2 = 0.$

Then the general expression is

$$E_y^\circ = \frac{j\omega\mu}{h^2}\,[-C_1\,C_3\,B\cos Ay\sin Bx - C_1\,C_4\,B\sin Bx\sin Ay]$$

If $x = a$, then $E_y^\circ = 0.$

$$E_y^\circ = \frac{-j\omega\mu}{h^2}\,[C_1\,C_3\,B\sin B\,a\cos A\,y + C_1\,C_4\sin B\,a\sin A\,y]$$

For $E_y^\circ = 0,\ B = \frac{m\pi}{a}.$

$$E_y^\circ = -\frac{j\omega\mu}{h^2}\,[\,C_1\,C_3\,B\sin Bx\cos Ay + C_1\,C_4\,B\sin Bx\sin Ay\,]$$

$$E_x^\circ = \frac{j\omega\mu}{h^2}\,[\,C_1\,C_3\,A\sin Ay\cos Bx + C_2\,C_3\,A\sin Ay\sin Bx\,]$$

Substituting the value $C_2 = C_4 = 0.$

$$E_x^\circ = \frac{j\omega\mu}{h^2}\,C_1\,C_3\,A\cos Bx\sin Ay$$

$$= \frac{j\omega\mu}{h^2}\,C_1\,C_3\,A\cos\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)y$$

$$E_y^\circ = -\frac{j\omega\mu}{h^2}\,C_1\,C_3\,B\sin Bx\cos Ay$$

$$= -\frac{j\omega\mu}{h^2}\,C_1\,C_3\,B\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y$$

Let $C = C_1\,C_3$

$$E_x^\circ = \frac{j\omega\mu}{h^2}\,C\,A\sin Ay\cos Bx$$

$$E_y^\circ = -\frac{j\omega\mu}{h^2}\,C\,B\sin Bx\cos Ay$$

where $A = \frac{n\pi}{b}$ and $B = \frac{m\pi}{a}$

Similarly for H_x°,

$$H_x^\circ = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \qquad [\because E_z = 0]$$

For propagation, $\gamma = j\beta$, $\quad [\because \alpha = 0]$

$$H_x^\circ = -\frac{j\beta}{h^2}\frac{\partial H_z}{\partial x}$$

But $\quad E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$

$$\frac{\partial H_z}{\partial x} = \frac{h^2}{j\omega\mu} \cdot E_y$$

Substituting the value of $\frac{\partial H_z}{\partial x}$ in the above H_x° equation

$$H_x^{\circ} = \frac{-j\beta}{h^2} \cdot \frac{h^2}{j\omega\mu} E_y^{\circ}$$

$$= \frac{-\beta}{\omega\mu} E_y^{\circ}$$

Substituting the value of E_y° in the above H_x° equation

$$H_x^{\circ} = \frac{-\beta}{\omega\mu}\left[\frac{-j\omega\mu}{h^2} \text{C B} \sin Bx \cos Ay\right]$$

$$H_x^{\circ} = \frac{j\beta}{h^2} \text{CB} \sin Bx \cos Ay$$

$$H_x^{\circ} = \frac{j\beta}{h^2} \text{CB} \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

Similarly for H_y°,

$$H_y^{\circ} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$= \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} \qquad [\because E_z = 0]$$

For propagation, $\gamma = j\beta$. $[\because \alpha = 0]$

$$H_y^{\circ} = \frac{-j\beta}{h^2}\frac{\partial H_z}{\partial y}$$

But $E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$

$$\frac{\partial H_z}{\partial y} = \frac{-h^2}{j\omega\mu} E_x$$

Substituting this value of $\frac{\partial H_z}{\partial y}$ in the above H_y° equation

$$H_y^{\circ} = \frac{-j\beta}{h^2}\frac{(-h^2)}{j\omega\mu} E_x^{\circ} = \frac{\beta}{\omega\mu} E_x^{\circ}$$

Substituting the value of E_x in the above H_y° equation

$$H_y^{\circ} = \frac{\beta}{\omega\mu}\left[\frac{j\omega\mu}{h^2} \text{C A} \sin Ay \cos Bx\right]$$

$$H_y^{\circ} = \frac{j\beta}{h^2} \text{C A} \cos B x \sin A y$$

$$H_y^{\circ} = \frac{j\beta}{h^2} \text{C A} \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$H_z^\circ = XY$$

$$= C_1 C_3 \cos Ay \cos Bx + C_2 C_3 \cos Ay \sin Bx + C_1 C_4 \cos Bx \sin Ay + C_2 C_4 \sin Ay \sin Bx$$

But $C_2 = C_4 = 0$

$$H_z^\circ = C_1 C_3 \cos Ay \cos Bx$$

$$C = C_1 C_3$$

$$H_z^\circ = C \cos Ay \cos Bx$$

$$H_z^\circ = C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

The field equations for TE waves are as follows :

$$H_x^\circ = \frac{j\beta}{h^2} CB \sin Bx \cos Ay$$

$$H_y^\circ = \frac{j\beta}{h^2} CA \cos Bx \sin Ay$$

$$H_z^\circ = C \cos Ay \cos Bx$$

$$E_x^\circ = \frac{j\omega\mu}{h^2} CA \cos Bx \sin Ay$$

$$E_y^\circ = \frac{-j\omega\mu}{h^2} CB \sin Bx \cos Ay$$

where $A = \frac{n\pi}{b}$ and $B = \frac{m\pi}{a}$

KONGUNADU COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRONICS AND COMMUNIATION ENGINEERING

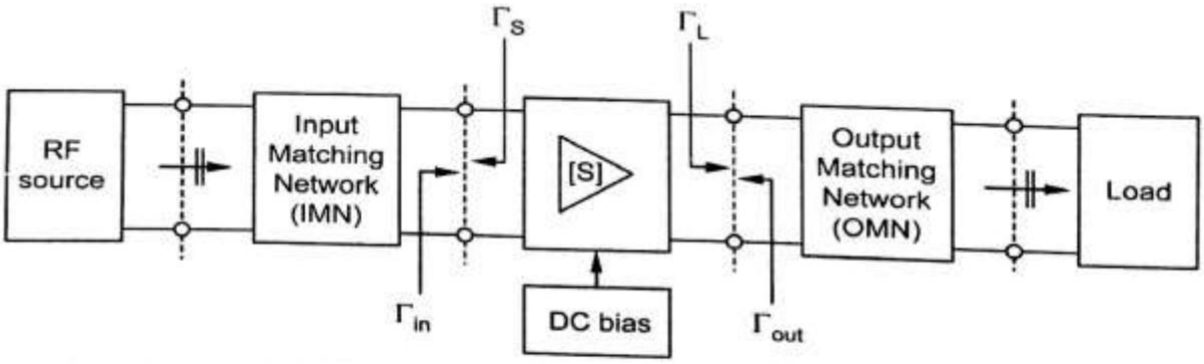
EC 8561 – TRANSMISSION LINES AND RF SYSTEMS

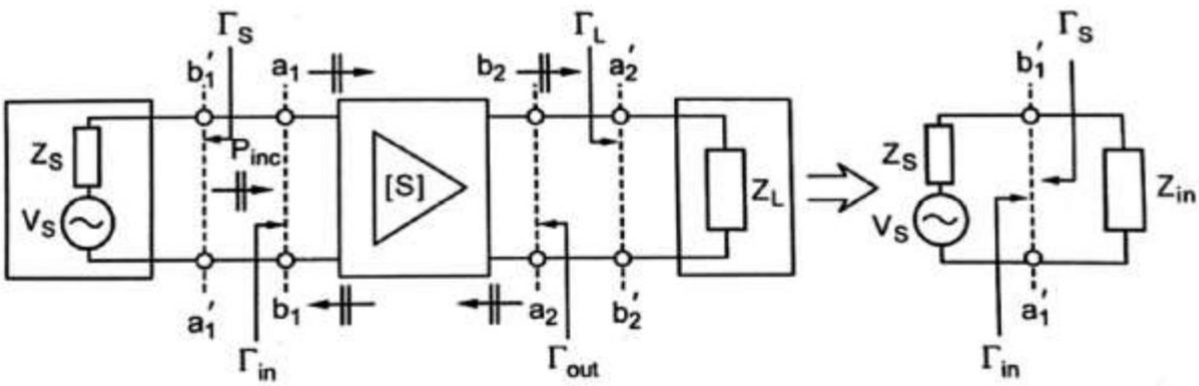
UNIT – V - RF SYSTEM DESIGN CONCEPTS

COURSE HANDOUTS

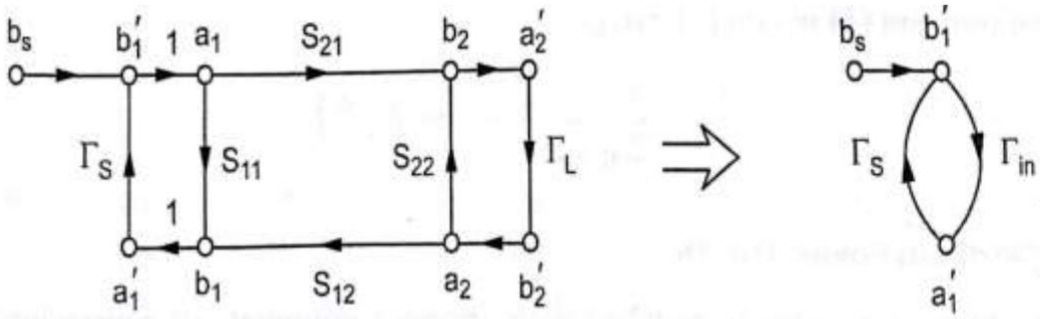
Amplifier Power Relation:

Generic single stage amplifier configuration with input and output matching networks is shown in fig.





(a) Simplified schematics of a single-stage amplifier



(b) Signal flow graph

RF source:

Incident Wave power:
The incident wave power at node 1′ is given by,

$\to 1$

Where, Source node

Input power:

$$P_{in} = P_{inc}\left(1 - |\Gamma_{in}|^2\right) \qquad P_{in} = \frac{1}{2}\frac{|b_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2}\left(1 - |\Gamma_{in}|^2\right)$$

Transducer power gain

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|S_{21}|^2\left(1 - |\Gamma_S|^2\right)}{|(1 - S_{11}\Gamma_S)(1 - S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_S|^2}$$

Unilateral power gain( ):

$$G_{TU} = \frac{\left(1 - |\Gamma_L|^2\right)|S_{21}|^2\left(1 - |\Gamma_S|^2\right)}{|1 - \Gamma_L S_{22}|^2|1 - S_{11}\Gamma_S|^2}$$

Additional power relations

Available Power Gain ( ) at Load:

$$G_A = \frac{|S_{21}|^2\left(1 - |\Gamma_S|^2\right)}{\left(1 - |\Gamma_{Out}|^2\right)|1 - S_{11}\Gamma_S|^2}$$

Power Gain (Operating Power Gain):

The operating power gain is defined as "the ratio of the power delivered to the load to the power supplied to the amplifier".

Stability considerations and Stabilization Methods:

- An amplifier circuit must be stable over the entire frequency range
- The RF circuits (amplifier) tend to oscillate depending on operating frequency and termination

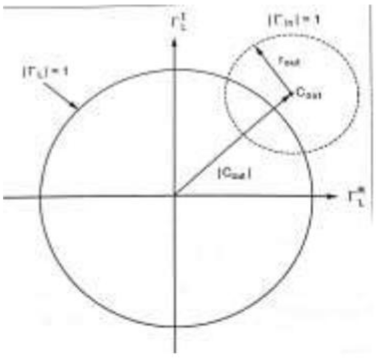
(i) If | | > 1, then the magnitude of the return voltage wave increases called positive feedback, which causes instability (oscillator)

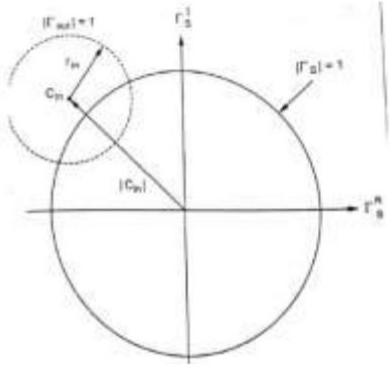
(ii) If | | < 1, the return voltage wave is totally avoided (amplifier). Its called as negative feedback

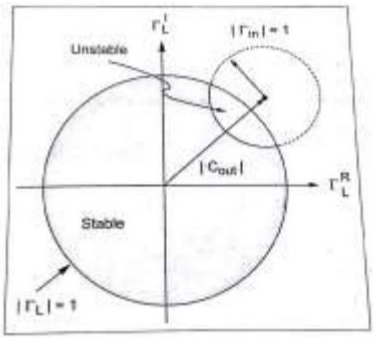
Two port network amplifier is characterized by its S-parameters
The amplifier is stable, when the magnitudes of reflection coefficients are less than unity

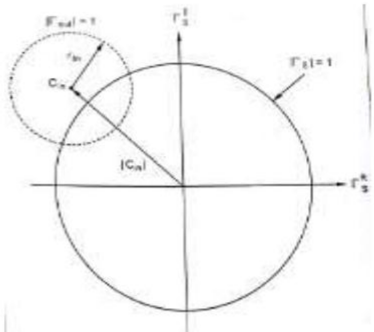
| | < 1 and

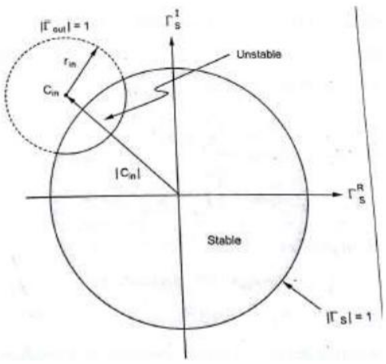
| |<1

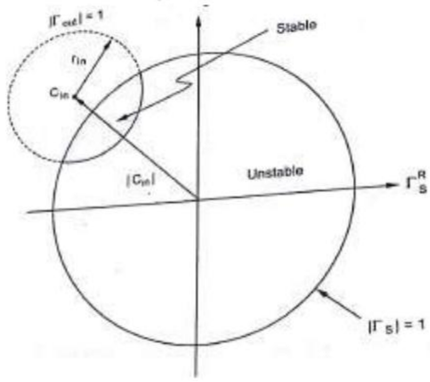












Stabilization Methods:

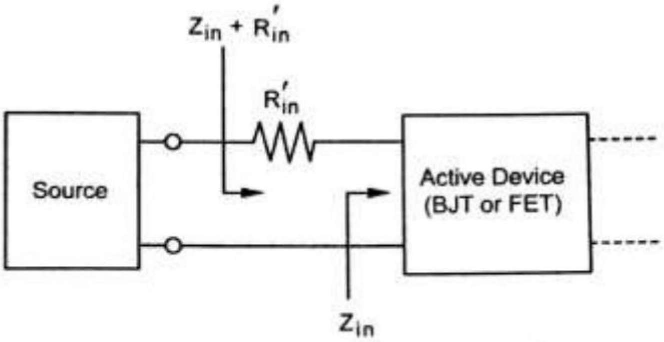
If the operation of a FET or BJT is unstable, we take steps to make them stable

The instability conditions | | > 1 and | | > 1 can be written in terms of the input and output impedances.

To stabilize the active devices, a series resistance or a conductance will be added to the port.

Configuration at input port:

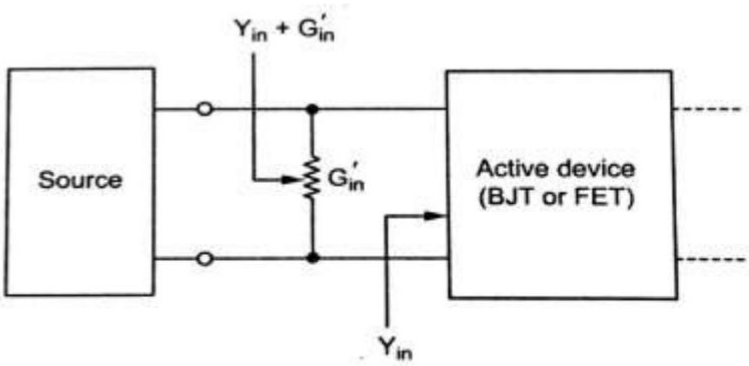
In the input port, the addition of ( ) must compensate the negative contribution of ( )



Stabilization of input port through series resistance

$$\mathrm{Re}(Z_{in} + R'_{in} + Z_S) > 0$$

Stabilization of input port through addition of shunt conductance.



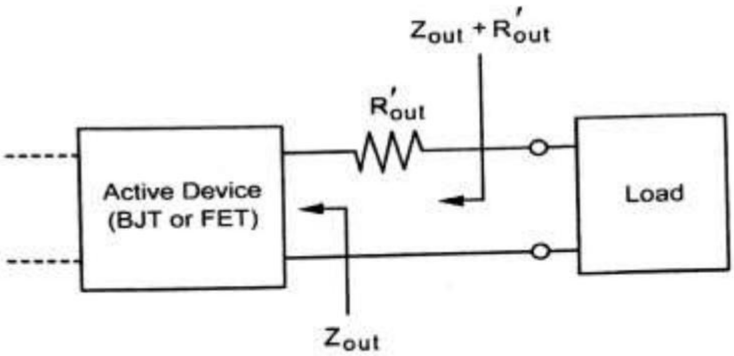
Stabilization of input port through shunt conductance

$$\mathrm{Re}(Y_{in} + G'_{in} + Y_S) > 0$$

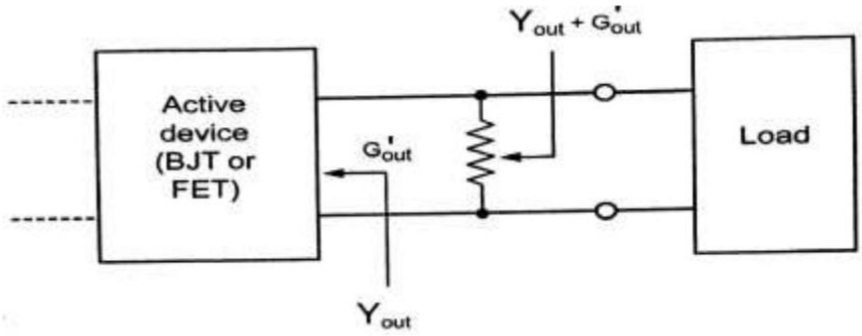
Configuration at output port:

In the output port, the addition of ( ) must compensate the negative contribution of ( )

$$\mathrm{Re}(Z_{out} + R'_{out} + Z_L) > 0$$



Stabilization of output port through series resistance



Stabilization of output port through shunt conductance

$$\mathrm{Re}(Y_{out} + G'_{out} + Y_L) > 0$$