UNIT I

RANDOM VARIABLE

PART – A (I Part)

1. A RV X has the following probability distribution

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

Find the value of k and mean of X.

Solution:

We Know that $\sum_{i=1}^{n} p(\mathrm{x_i}) = 1$

$$0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$$
$$\Rightarrow 6k + 0.6 = 1 \Rightarrow 6k = 1 - 0.6$$
$$\Rightarrow 6k = 0.4 \Rightarrow k = \frac{0.4}{6} = \frac{1}{15}$$

Mean $= E(\mathrm{X}) = \sum x_i\, p(\mathrm{x_i})$

$$= (-2)0.1 + (-1)\left(\frac{1}{15}\right) + 0 + 1\left(2\times\frac{1}{15}\right) + 2(0.3) + 3\left(3\times\frac{1}{15}\right) = \frac{16}{15}$$

2. If the RV X takes values 1, 2, 3 & 4 such that $2P(\mathrm{X}=1) = 3P(X=2) = P(X=3) = 5P(X=4)$ **find the probability distribution of X.**

Solution:

Assume P(X=3) = α by the given equation

$$P(X=1) = \frac{\alpha}{2},\ P(X=2) = \frac{\alpha}{3} \ \ \&\ P(X=4) = \frac{\alpha}{5}$$

WKT $\sum_{i=1}^{n} p(\mathrm{x_i}) = 1$

$$\Rightarrow \frac{\alpha}{2} + \frac{\alpha}{3} + \alpha + \frac{\alpha}{5} = 1$$
$$\Rightarrow \frac{61}{30}\alpha = 1$$
$$\Rightarrow \alpha = \frac{30}{61}$$
$$\therefore P(X=1) = \frac{15}{61};\ P(X=2) = \frac{10}{61};\ P(X=3) = \frac{30}{61};\ P(X=4) = \frac{6}{61}$$

The probability distribution is given by

X	1	2	3	4
$p(x)$	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

3. **A random variable X has the following probability function.**

Values of X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Probability p(x)	**a**	**3a**	**5a**	**7a**	**9a**	**11a**	**13a**	**15a**	**17a**

Determine the value of 'a'.

Solution:

$$\text{WKT } \sum_{i=1}^{n} p(x_i) = 1$$

$$\Rightarrow a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$\Rightarrow 81a = 1 \Rightarrow a = \frac{1}{81}$$

4. **A random variable X has the following probability function**

X	**0**	**1**	**2**	**3**	**4**
P(x)	**K**	**3k**	**5k**	**7k**	**9k**

Find the value of k.
Solution:

$$\text{WKT } \sum_{i=1}^{n} p(x_i) = 1$$

$$k + 3k + 5k + 7k + 9k = 1$$

$$25k = 1 \Rightarrow k = \frac{1}{25}$$

5. **Show that the function** $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0 & ow \end{cases}$ **is PDF.**

Solution:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{2} \frac{x^2}{3}\,dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2} = \frac{1}{9}(8+1) = 1$$

6. **If X is a continuous random variable whose probability density function is given by** $f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & ow \end{cases}$ **what is the value of 'c'?**

Solution:

$$\text{WKT } \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{2} c(4x - 2x^2)\,dx = 1 \Rightarrow 2c\int_{0}^{2} (2x - x^2)dx = 1$$

$$2c\left[\frac{2x^2}{2}-\frac{x^3}{3}\right]_0^2=1 \Rightarrow 2c\left[4-\frac{8}{3}\right]=1$$

$$2c\left[\frac{4}{3}\right]=1 \Rightarrow c=\frac{3}{8}$$

7. A continuous random variable X has a pdf $f(x)=k,\ 0\le x\le 1$. Find 'k'.

Solution:

WKT $\int_{-\infty}^{\infty} f(x)\,dx=1$

$$\int_0^1 k\,dx=1 \Rightarrow k\left[x\right]_0^1=1 \Rightarrow k(1-0)=1 \Rightarrow k=1$$

8. If a random variable X has the pdf $f(x)=\begin{cases}\frac{1}{4}, & |x|<2\\ 0, & ow\end{cases}$. Find $P(X<1)$

Solution:

Given $f(x)=\begin{cases}\frac{1}{4}, & |x|<2\\ 0, & ow\end{cases} \Rightarrow f(x)=\begin{cases}\frac{1}{4}, & -2<x<2\\ 0, & ow\end{cases}$

$$\therefore P(X<1)=\int_{-\infty}^{1} f(x)\,dx=\int_{-2}^{1}\frac{1}{4}dx=\frac{1}{4}\left[x\right]_{-2}^{1}=\frac{1}{4}\left[1+2\right]=\frac{3}{4}$$

9. A random variable X has the pdf $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & ow\end{cases}$ find $P\left(X<\frac{1}{2}\right)$

Solution:

$$P\left(X<\frac{1}{2}\right)=\int_0^{1/2} 2x\,dx=2\left[\frac{x^2}{2}\right]_0^{\frac{1}{2}}=\left[\frac{1}{4}-0\right]=\frac{1}{4}$$

10. A continuous random variable X has a pdf $f(x)=3x^2,\ 0\le x\le 1$. Find 'b' such that $P(X>b)=0.05$.

Solution:

Given $f(x)=3x^2,\ 0\le x\le 1$

When $P(X>b)=0.05$

$$\Rightarrow \int_b^1 f(x)\,dx=0.05 \Rightarrow \int_b^1 3x^2\,dx=0.05$$

$$\Rightarrow 3\left[\frac{x^3}{3}\right]_b^1=0.05 \Rightarrow 1-b^3=0.05 \Rightarrow b^3=1-0.05 \Rightarrow b=0.9830$$

11. In a continuous random variable X having the pdf $f(\text{x}) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & ow \end{cases}$**.Find**

$P(0 < \text{x} \le 1)$

Solution:

$$P(0 < \text{x} \le 1) = \int_0^1 f(\text{x})\,dx = \int_0^1 \frac{x^2}{3}\,dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}\left[\frac{1}{3}\right] - 0 = \frac{1}{9}$$

12. A random variable X has the density function $f(\text{x}) = \text{k}\frac{1}{1+x^2}$ **in** $-\infty < x < \infty$**. Find 'k'.**

Solution:

$$\text{WKT} \int_{-\infty}^{\infty} f(\text{x})\,dx = 1 \Rightarrow k\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 1 \Rightarrow 2k\int_0^{\infty} \frac{1}{1+x^2}\,dx = 1$$

$$2k\left[\tan^{-1} x\right]_0^{\infty} = 1 \Rightarrow 2k\left[\frac{\pi}{2} - 0\right] = 1 \Rightarrow k(\pi) = 1 \Rightarrow \text{k} = \frac{1}{\pi}$$

13. For the following CDF $F(\text{x}) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$ **find** (i) $\text{P}(\text{X} > 0.2)$ **and** (ii) $\text{P}(0.2 < \text{x} \le 0.5)$

Solution:

(i) $\text{P}(\text{X} > 0.2) = 1 - P(\text{X} \le 0.2) = 1 - \text{F}(0.2) = 1 - 0.2 = 0.8$ and

(ii) $\text{P}(0.2 < \text{x} \le 0.5) = F(0.5) - \text{F}(0.2) = 0.5 - 0.2 = 0.3$

14. The density function of a RV X is given by $f(x) = kx(2-x)$**,** $0 \le x \le 2$ **find the value of k.**

Solution:

Given $f(x) = kx(2-x)$, $0 \le x \le 2$

$$\text{WKT} \int_{-\infty}^{\infty} f(\text{x})\,dx = 1 \Rightarrow \int_0^2 k\,x(2-\text{x})\,dx = 1 \Rightarrow k\int_0^2 (2\text{x} - \text{x}^2)\,dx = 1$$

$$\Rightarrow k\left[2\frac{x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1 \Rightarrow k\left[2^2 - \frac{2^3}{3}\right] = 1 \Rightarrow k\left[4 - \frac{8}{3}\right] = 1$$

$$\Rightarrow k\left[\frac{4}{3}\right] = 1 \Rightarrow k = \frac{3}{4}$$

15. A continuous RV X has the PDF $f(x) = k(1+x)$, $2 \le x \le 5$ find $P(X < 4)$.

Solution:

Given $f(x) = k(1+x)$, $2 \le x \le 5$

$$\text{WKT} \int_{-\infty}^{\infty} f(\text{x})\,dx = 1$$

$$\Rightarrow \int_2^5 k(1+\text{x})\,dx = 1 \Rightarrow k\left[x + \frac{x^2}{2}\right]_2^5 = 1 \Rightarrow k\left[5 + \frac{25}{2} - 2 - \frac{4}{2}\right] = 1$$

$$\Rightarrow k\left[3+\frac{21}{2}\right]=1\Rightarrow k\left[\frac{27}{2}\right]=1\Rightarrow k=\frac{2}{27}$$

And $P(X<4)=\int_{-\infty}^{4} f(x)\,dx=k\int_{2}^{4}(1+x)\,dx=k\left[x+\frac{x^2}{2}\right]_2^4=k\left[4+8-(2+2)\right]=8k=\frac{16}{27}$

16. Find the MGF for the distribution where $f(x)=\begin{cases}\frac{2}{3} & at\ x=1\\ \frac{1}{3} & at\ x=2\\ 0 & ow\end{cases}$

Solution:

Given $f(1)=\frac{2}{3}$; $f(2)=\frac{1}{3}$ & $f(3)=f(4)=...=0$

$$M_X(t)=E\left[e^{tx}\right]=\sum_{X=0}^{\infty}e^{tx}\,f(x)=e^0f(0)+e^1f(1)+e^2f(2)+...$$

$$=0+e^t\left(\frac{2}{3}\right)+e^{2t}\left(\frac{1}{3}\right)+0\ =\frac{2e^t}{3}+\frac{e^{2t}}{3}=\frac{e^t}{3}\left[2+e^t\right]$$

17. If a RV X has the MGF $M_X(t)=\frac{2}{2-t}$ **find Var (X).**

Solution:

Given $M_X(t)=\frac{2}{2-t}=2(2-t)^{-1}$

$$M_X'(t)=-2(2-t)^{-2}(-1)=2(2-t)^{-2}$$

$$M_X''(t)=-4(2-t)^{-3}(-1)=4(2-t)^{-3}$$

WKT $E\left[X\right]=M_X'(0)=2(2-0)^{-2}=2\times 2^{-2}=\frac{1}{2}$

$$\&\ E\left[X^2\right]=M_X''(0)=4(2-0)^{-3}=4\times 2^{-3}=\frac{1}{2}$$

$$\therefore Var(X)=E\left[X^2\right]-\left[E\left[X\right]\right]^2=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

18. Let X be a RV with $E(X)=10$ **and** $Var(X)=25$ **Find the positive values of a and b such that** $Y=aX-b$ **has expectation 0 and variance 1.**

Solution:

Given $E\left[X\right]=10$, $Var(X)=25$ and $E\left[Y\right]=0$, $Var\left[Y\right]=1$

Given $Y=aX-b\Rightarrow E\left[Y\right]=aE\left[X\right]-b\Rightarrow 0=a(10)-b\Rightarrow 10a=b$

$$\&\ Var\left[Y\right]=a^2\,Var\left[X\right]\Rightarrow 1=a^2(25)\Rightarrow a^2=\frac{1}{25}\Rightarrow a=\frac{1}{5}$$

$$\therefore b=10\times\frac{1}{5}\Rightarrow b=2$$

19. State and prove additive property of binomial distribution.

Solution:

The sum of two binomial variates is not a binomial variate.

Let X and Y be two independent binomial variates with parameter (n_1, p_1) and (n_2, p_2) respectively.

Then $M_X(t) = (q_1 + p_1 e^t)^{n_1}$ and $M_Y(t) = (q_2 + p_2 e^t)^{n_2}$

$\therefore M_{X+Y}(t) = M_X(t).M_Y(t) = (q_1 + p_1 e^t)^{n_1}.(q_2 + p_2 e^t)^{n_2} \neq (q + pe^t)^n$

Hence by uniqueness theorem of MGF X + Y is not a binomial variate.

20. Check whether the following data follow a binomial distribution or not Mean = 3; Variance = 4.

Solution:

Given Mean = $np = 3$ and Variance = $npq = 4$

$\therefore \frac{npq}{np} = \frac{4}{3} \Rightarrow q = \frac{4}{3} > 1$ Which is not possible.

The given data dose not follow Binomial distribution.

21. If the probability is 0.05 that a certain kind of measuring device will show excessive drift, what is the probability that the sixth of these measuring devices tested will be the first to show excessive drift?

Solution:

Here $p = 0.05 \Rightarrow q = 1 - p = 1 - 0.05 = 0.95$ and Given $x = 6$

$\therefore P(X = x) = q^{x-1}p = (0.05)(0.95)^5 = 0.0387$

22. If 3% of the electric bulbs manufactured by a company are defective, find the probability that in the sample of 100 bulbs exactly 5 bulbs are defective. $(e^{-3} = 0.0498)$

Solution:

Let X be the RV denoting the number of defective electric bulbs.

Given P (a bulb is defective) $= \frac{3}{100}$

$\Rightarrow p = 0.03$ and $n = 100$

$\therefore$ Mean = $\lambda = np = 100 \times 0.03 = 3$

WKT $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\therefore$ P (exactly 5 bulbs are defective) $= P(X = 5) = \frac{e^{-3}3^5}{5!} = \frac{0.0498 \times 243}{120} = 0.1008$

23. The time (in hours) required to repair a machine is exponentially distributed with parameter what is the probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours.

Solution:

Let X be the RV which represents the time to repair the machine.

$P(X > 10 / X > 9) = P(X > 1)$

$= P(X > 9 + 1 / X > 9)$ [by memory less property]

$= e^{-\frac{1}{2}} = 0.6065 \quad \left[\because P(X > k) = e^{-\lambda t} \ \ here\ \lambda = \frac{1}{2}\ ,\ t = 1 \right]$

24. A normal distribution has mean and S. D. Find $P(15 \le X \le 40)$

Solution:

Given $\mu = 20$ and $\sigma = 10$

The normal variate $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 20}{10}$

When $X = 15 \Rightarrow Z = \dfrac{15 - 20}{10} = -0.5$

When $X = 40 \Rightarrow Z = \dfrac{40 - 20}{10} = 2$

$\therefore P(15 \le X \le 40) = P(0.5 \le Z \le 2) = P(-0.5 \le Z \le 0) + P(0 \le Z \le 2)$

$= P(0 \le Z \le 0.5) + P(0 \le Z \le 2) = 0.1915 + 0.4772 = 0.6687$

PART – B

1. A RV X has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2$+a

Find (i) a (ii) $P(X < 4)$ (iii) $P(X \ge 4)$ (iv) $P(X \le k) > 1/2$ find the least value of k. Find the CDF of X.(v) $P(1.5 < X < 4.5 / X > 2)$ (iv) $P(X < 2)$, $P(X > 3)$, $P(1 < X < 5)$

2. Find the cumulative distribution function of the RV with pdf $f(x) = \begin{cases} x & 0 < x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & ow \end{cases}$ and find the mean, variance and MGF of X.

3. The amount of time in hours that a computer function before breaking down is a Continuous RV with pdf given by $f(x) = \begin{cases} \lambda e^{-x/100} & x \ge 0 \\ 0 & x < 0 \end{cases}$ what is the probability that (i) a computer will

function between 50 and 150 hours before breaking down. (ii) it will function less than 500 hours.

4. A RV X has the pdf $f(x)=\begin{cases}2x & 0<x<1\\ 0 & ow\end{cases}$ find (i) $P(X<1/2)$ (ii) $P(1/4<X<1/2)$ (iii) $P(X>3/4/X>1/2)$ (iv) $P(X<3/4/X>1/2)$

5. Verify whether $f(x)=\begin{cases}\frac{1}{2}(x+1) & -1<x<1\\ 0 & ow\end{cases}$ is a probability function of a continuous RV X.

If so, find the mean and Var(X).

6. The distribution function of a RV is given by $f(x)=\begin{cases}kx(2-x) & 0\le x\le 2\\ 0 & ow\end{cases}$ find the value of k, the mean, variance and rth moment.

7. Let the RV X have pdf $f(x)=\frac{1}{2}e^{-x/2}, x>0$ find the MDF and hence find the mean and variance of X.

8. A RV X has the pdf given by $f(x)=\begin{cases}2e^{-2x} & x\ge 0\\ 0 & x<0\end{cases}$ find (i) the MGF (ii) the first 4 moments about the origin.

9. A Discrete RV X has the following probability distribution

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i)Find the value of a. (ii) $P(X<3)$, $P(0<X<3)$, $P(X\ge 3)$
(iii) Find distribution function of X.

10. Find the MGF of the RV X whose probability function $P(X=x)=\frac{1}{2^x}$, $x=1,2,3...$ hence find its mean.

11. A RV X has the following probability distribution

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

(i) Find the value of k (ii) $P(X<2)$ and $P(-2<X<2)$ (iii) Find the CDF of X
(iv) mean of X.

12. Find the MGF of the Binomial Distribution and hence find the mean and variance.

13. A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws what is the probability that there are (i) Exactly 3 defectives (ii) not more than 3 defectives.

14. Derive the Poisson distribution as limiting form of Binomial distribution.

15. Find the MGF of the Poisson distribution and find the mean and variance.

16. State and prove the additive property of Poisson distribution.

17. The number of monthly breakdown of a computer is a RV having a poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month. (i) Without break down (ii) with only one break down (iii) with atleast one breakdown.

18. The probability that an individual suffers from a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals (i) exactly 3 (ii) more than 2, individuals will suffer from a bad reaction.

19. If X_1 and X_2 are independent variate with parameters λ_1 & λ_2 show that the conditional distribution of X_1 given $X_1 + X_2 = n$ follows Binomial distribution.

20. Prove that the geometric RV has memory less property.

21. Find the MGF, mean and variance of uniform distribution.

22. A RV X has a uniform distribution over (-3, 3), compute (i) $P(X < 2)$ (ii) $P(|X| < 2)$ (iii) $P(|X-2| < 2)$ (iv)Find k for which $P(X > k) = 1/3$

23. If X is uniformly distributed over $(-\alpha, \alpha)$ find α so that (i) $P(X > 1) = 1/3$ (ii) $P(|X| < 1) = P(|X| > 1)$

24. Find the MGF of the Exponential distribution, mean and variance.

25. State and prove memoryless property for Exponential distribution.

26. The time required to repair a machine is Exponential distribution with parameter $\lambda = 1/2$ (i) what is the probability that the repair time exceeds 2 hours? (ii) What is the conditional probability that a repair takes 11 hours? Given that its duration exceeds 8 hours?

27. The mileage which can owners get with certain kind a radial tyre is a RV having an Exponential distribution with mean 4000 km. Find the probability that one of these tyres will last (i) atleast 2000 km (ii) atmost 3000 m.

28. An electrical firm manufactures light bulbs that have a life, before burn – out, that is normally distributed with mean equal to 800 hours and a SD of 40 hours. Find (i) the probability that a bulb more than 834 hours. (ii) the probability that a bulb between 778 and 834 hours.

UNIT II

TWO DIMENSIONAL RANDOM VARIABLES

PART – A

1. **The joint pdf of random variables X and Y is given by $f(x,y) = k\,xy e^{-(x^2+y^2)}$ $x > 0, y > 0$. Find the value of k .**
Solution:

WKT $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathrm{x},\mathrm{y})\,dy\,dx = 1$

Given $f(x,y) = k\,xy\,e^{-(x^2+y^2)}\ x>0, y>0$

$$\therefore \int_0^\infty\int_0^\infty k\,xy\,e^{-(x^2+y^2)}dy\,dx = 1 \Rightarrow k\int_0^\infty\int_0^\infty xy\,e^{-x^2}e^{-y^2}dy\,dx = 1$$

$$\Rightarrow k\int_0^\infty x\,e^{-x^2}dx\int_0^\infty y\,e^{-y^2}dy = 1 \Rightarrow k.\frac{1}{2}.\frac{1}{2} = 1 \Rightarrow k = 4$$

2. **If X and Y have joint pdf of** $f(x,y) = \begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & ow \end{cases}$ **check whether X and Y are independent.**

Solution:

To prove: $f_X(\mathrm{x})\,\mathrm{f_Y(y)} = \mathrm{f(x,y)}$

$$\mathrm{f_X(x)} = \int_0^\infty f(\mathrm{x},\mathrm{y})\,dy = \int_0^1 (x+y)\,dy = \left[xy + \frac{y^2}{2}\right]_0^1 = x + \frac{1}{2}$$

$$\mathrm{f_Y(y)} = \int_0^\infty f(\mathrm{x},\mathrm{y})\,dx = \int_0^1 (x+y)\,dx = \left[\frac{x^2}{2} + xy\right]_0^1 = \frac{1}{2} + y$$

$$\therefore f_X(\mathrm{x})\,\mathrm{f_Y(y)} = \left(x+\frac{1}{2}\right)\left(\frac{1}{2}+y\right) \neq x+y \neq f(\mathrm{x},\mathrm{y})$$

$\Rightarrow$ X and Y are not independent.

3. **Find k of the joint pdf of a bivariant random variable (X, Y) is given by**

$$f(x,y) = \begin{cases} k(1-x)(1-y) & 0<x<4,\ 1<y<5 \\ 0 & ow \end{cases}$$

Solution:

WKT $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathrm{x},\mathrm{y})\,dy\,dx = 1$

$$\therefore \int_0^4\int_1^5 k(1-\mathrm{x})(1-\mathrm{y})\,dy\,dx = 1 \Rightarrow k\int_0^4 (1-\mathrm{x})\,dx\int_1^5 (1-\mathrm{y})\,dy = 1$$

$$\Rightarrow k\left\{\left(x - \frac{x^2}{2}\right)_0^4\left(y - \frac{y^2}{2}\right)_1^5\right\} = 1 \Rightarrow k = \frac{1}{32}$$

4. **The random variable (X, Y) has the pdf** $f(x,y) = kx^2(8-y), x<y<2x, 0\le x<2$ **. Find k.**

Solution:

WKT $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathrm{x},\mathrm{y})\,dy\,dx = 1$

$$\therefore \int_0^2\int_x^{2x} k\,x^2(8-\mathrm{y})\,dy\,dx = 1 \Rightarrow k\int_0^2 x^2\left(8y - \frac{y^2}{2}\right)_x^{2x} dx = 1$$

$$\Rightarrow k\int_0^2 x^2\left(16x-\frac{4x^2}{2}-8x+\frac{x^2}{2}\right)dx=1$$

$$k\int_0^2\left(16x^3-2x^4-8x^3+\frac{x^4}{2}\right)dx=1\Rightarrow k\int_0^2\left(8x^3-\frac{3x^4}{2}\right)dx=1$$

$$k\left[\frac{8x4}{4}-\frac{3}{2}\frac{x5}{5}\right]_0^2=1\Rightarrow k\left[32-\frac{48}{5}\right]=1\Rightarrow k\left(\frac{112}{5}\right)=1\Rightarrow k=\frac{5}{112}$$

5. If the joint pdf of (X,Y) is $f(x,y)=\begin{cases}\frac{1}{4} & 0<x,y<2\\ 0 & ow\end{cases}$ **, Find** $P(X+Y\le 1)$

Solution:

$$P(X+Y\le 1)=\int_0^1\int_0^{1-y} f\,(\text{x},\text{y})\,dx\,dy$$

$$=\int_0^1\int_0^{1-y}\frac{1}{4}dx\,dy=\frac{1}{4}\int_0^1(\text{x})_0^{1-y}dy=\frac{1}{4}\int_0^1(1-\text{y})\,dy$$

$$=\frac{1}{4}\left[y-\frac{y2}{2}\right]_0^1=\frac{1}{4}\left[1-\frac{1}{2}\right]=\frac{1}{8}$$

6. The joint pdf of X and Y is given by $P(x,y)=\frac{x+y}{21},\ x=1,2,3;\ y=1,2$ **. Find the marginal probability distribution of X, Y.**

Solution:

The marginal distribution are given in the table

Y / X	1	2	3	Marginal distribution of Y = $P_Y(\text{y})$
1	2 / 21	3 / 21	4 / 21	9 / 21
2	3 / 21	4 / 21	5 / 21	12 / 21
Marginal distribution of X= $P_X(\text{x})$	5 / 21	7 / 21	9 / 21	1

7. The joint probability mass function of X and Y is

	0	**1**	**2**
0	**0.1**	**0.04**	**0.02**
1	**0.08**	**0.20**	**0.06**
2	**0.06**	**0.14**	**0.30**

Find the marginal functions of X and Y, $P(X\le 1,Y\le 1)$ **and check whether X and Y are independent.**

Solution:

The Marginal distribution are given in the table

X/Y	0	1	2	Marginal distribution of X = $P_X(\text{x})$
0	0.10	0.04	0.02	0.16

1	0.08	0.20	0.06	0.34
2	0.06	0.14	0.30	0.50
Marginal distribution of Y= $P_Y(y)$	0.24	0.38	0.38	1

(i) $P(X \le 1, Y \le 1) = P(0,0) + P(1,0) + P(0,1) + P(1,1) = 0.1 + 0.08 + 0.04 + 0.2 = 0.42$

(ii) Here $P(X=0)P(Y=0) = 0.16 \times 0.24 \ne 0.1 \ne P(X=0, Y=0)$

$\therefore$ X and Y are not independent.

8. The following data were available $\bar{X} = 970, \bar{Y} = 18, \sigma_X = 38, \sigma_Y = 2$ correlation coefficient r = 0.6. Find the line of regression and obtain the value of X given Y = 20.

Solution:

WKT the line of regression of X on Y is given by $X - \bar{X} = r.\frac{\sigma_X}{\sigma_Y}(Y - \bar{Y})$

Given $\bar{X} = 970, \bar{Y} = 18, \sigma_X = 38, \sigma_Y = 2$

$\therefore X - 970 = 0.6 \times \frac{38}{2}(Y - 18) = 11.4Y - 205.2$

$i.e).,$ $x = 11.4y + 764.8$ This gives the lime of regression of X onY

When $y = 20 \Rightarrow x = 11.4(20) + 764.8 \Rightarrow x = 992.8$

9. State Central limit theorem.

Solution:

If $X_1, X_2, \ldots X_n \ldots$ be a sequence of independent identically distributed Random Variable with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$, $i = 1, 2, 3\ldots$ and if $S_n = X_1 + X_2 + X_3 + \ldots X_n$, then S_n follows a normal distribution with mean $n\mu$ and variance $n\sigma^2$ as $n \to \infty$.

10. State the equation of the two regression lines. What is the angle between them?

Solution:

The line of regression of X on Y is $X - \bar{X} = b_{XY}(Y - \bar{Y})$

The line of regression of Y on X is $Y - \bar{Y} = b_{YX}(X - \bar{X})$

The angle between the two lines of regression is given by $\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma_Y \sigma_X}{\sigma_{X^2} + \sigma_{Y^2}}\right)$

11. If X and Y are independent random variables with variance 2 and 3. Find the variance of $3X + 4Y$

Solution:

Given $Var(X) = 2$ and $Var(Y) = 3$

$\therefore Var(3X + 4Y) = 3^2\,Var(X) + 4^2\,Var(Y) = 9 \times 2 + 16 \times 3 = 18 + 48 = 66$

12. If the joint pdf of (X,Y) is given by $f(x,y) = x + y \;\; 0 \le x, y \le 1$ Find $E[XY]$

Solution:

Given $f(x,y) = x + y \;\; 0 \le x, y \le 1$

Now $E[XY]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\,dy\,dx=\int_0^1\int_0^1 xy\,(x+y)\,dy\,dx=\frac{1}{3}$

PART- B

1. The joint probability mass function of (X, Y) is given by $p(x,y)=k(2x+3y), x=0,1,2; y=1,2,3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of X + Y and $P(X+Y>3)$.

2. The joint probability density function of two dimensional random variable (X, Y) is given by $f(x,y)=xy^2+\frac{x^2}{8},\ 0\le x\le 2, 0\le y\le 1$ Compute $P(X>1), P(Y<1/2),\ P\left(X>1/Y<\frac{1}{2}\right)$, $P\left(Y<\frac{1}{2}/X>1\right)$, $P(X<Y)$ and $P(X+Y\le 1)$.

3. The random variables X and Y have joint probability density function $f(x,y)=\begin{cases} x^2+\frac{xy}{3} & 0<x<1,\ 0\le y\le 2 \\ 0 & ow \end{cases}$, then find (i) the marginal density function of X and Y (ii) the conditional density function of X given Y (iii) Are X and Y independent?

4. The joint probability mass function of X and Y is given below

	-1	1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient of X and Y.

5. The random variable X and Y have joint density function $f(x,y)=\begin{cases} 2-x-y, & 0<x<1,\ 0<y<1 \\ 0 & ow \end{cases}$ Find Cov (X, Y) and correlation coefficient of X and Y.

6. The resistors R_1, R_2, R_3, R_4 are independent random variable and uniform in the interval (450, 550). Using central limit theorem find $P(1900\le R_1, R_2, R_3, R_4\le 2100)$

7. A fair coin is tossed 300 times. What is the probability that head will appear more than 140 times and less than 150 times.

8. The life time of a certain brand of tube light may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Find the probability using central limit theorem, that the average life time of 60 lights exceeds 1250 hours.

9. A random sample of size 100 is taken from a population whose mean is 60 and variance 400 using central limit theorem, with what probability can we assert that the mean of the sample will not differ from $\mu=60$ by more than 4.

10. If the joint pdf of (X, Y) is given by $f(x, y) = x + y, 0 \le x, y \le 1$, find the pdf of U = XY.

11. Let (X, Y) be a two – dimensional non – negative continuous random variable having the joint density $f(x, y) = \begin{cases} 4xy e^{-(x^2+y^2)}, & x \ge 0, \ y \ge 0 \\ 0 & ow \end{cases}$ Find the density function of $U = \sqrt{X^2 + Y^2}$

12. If X and Y independent random variable with pdf $e^{-x}, \ x \ge 0$ and $e^{-y}, \ y \ge 0$, find the density function of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are they independent?

13. The joint pdf of X and Y is given by $f(x, y) = e^{-(x+y)}, x > 0, \ y > 0$, find the probability density function of $U = \frac{X+Y}{2}$.

14. Calculate the correlation coefficient for the height (in inches) of father (X) and their son (Y)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

15. From the following data, find (i) the coefficient of correlation between the marks in Economics and Statistics.
(ii) The two regression equations. (iii) the most likely marks in statistics when the marks in Economics are30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

16. The following table gives the joint pdf of two random variables X and Y. Find E(X), E(Y) and E (XY). Verify whether X and Y are correlated.

	0	1	2	3
2	1/8	1/8	1/8	1/8
3	1/16	1/8	0	1/16
4	1/16	0	1/8	1/16

17. If the joint cumulative distribution function of X and Y is given by
$$F(x, y) = \begin{cases} (1-e^{-x})(1-e^{-y}), & x > 0, y > 0 \\ 0 & ew \end{cases}$$
(i) Find the marginal density function of X and Y
(ii) Are X and Y independent?
(iii) Find $P(1 < X < 2, \ 1 < Y < 2)$

Given $f_{XY}(x, y) = \begin{cases} cx(x-y), & 0 < x < 2, \ -x < y < x \\ 0 & ow \end{cases}$ (a) Evaluate c. (b) Find $f_X(x)$ (c) $f_{X/Y}\left(\frac{y}{x}\right)$

and (d) $f_Y(y)$

19. Given the joint pdf of (X, Y) as $f(x,y)=\begin{cases}8xy, & 0<x<y<1\\ 0 & ow\end{cases}$ Find the marginal and conditional probability
function of X and Y. Are X and Y independent? Find $P(X+Y>1)$.

20. The two dimensional random variable (X, Y) has the joint probability mass function $f(x,y)=\dfrac{x+y}{27},\ x=0,1,2;\ y=0,1,2$ (i) Find the conditional distribution of Y given X = x.
(ii) Also find the conditional distribution of Y given X = 1.

UNIT III
CLASSIFICATION OF RANDOM PROCESSES
PART - A

1. **Define WSS process.**
Solution:
(i) $E\left[X(\mathrm{t})\right]=\text{Constant}$ and (ii) $E\left[X(\mathrm{t})\,\mathrm{X}(\mathrm{t}+\tau)\right]=R_{XX}(\tau)$

2. **Define SSS process.**
Solution:
(i) $E\left[X(\mathrm{t})\right]=\text{Constant}$ and (ii) $Var\left[X(\mathrm{t})\right]=\text{Constant}$

3. **What is Markov process and Markov Chain?**
Solution:
If the future value depends only on the present state but not on the past states is called a Markov process.
A discrete parameter Markov process is called a Markov Chain.
i.e.)., $P\left[X_n=a_n/x_{n-1}=a_{n-1}\right]\ \forall n$

4. **Define a Poisson process. (Or) state the postulates of a Poisson process.**
Solution:
If X (t) represents the number of occurrences of a certain event (0, t), then the discrete random process is called the Poisson, Provided that the following postulates are satisfied.
(i) $P\left[1\ \text{occurrence in}\ (\mathrm{t},\mathrm{t}+\Delta\,\mathrm{t})\right]=\lambda\Delta t+o(\Delta\,\mathrm{t})$
(ii) $\mathrm{P}\left[0\ \text{occurence in}\ (\mathrm{t},\mathrm{t}+\Delta\,\mathrm{t})\right]=1-\lambda\Delta t+o(\Delta\,\mathrm{t})$
(iii) $P\left[2\ \text{or more occurences in}\ (\mathrm{t}+\mathrm{t}+\Delta\,\mathrm{t})\right]=o(\Delta\,\mathrm{t})$

5. **If the tpm of a Markov chain is $\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$, find the steady state distribution of the chain.**

Solution:
Given $P=\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$
The invariant probability distribution is the steady state distribution.
$\therefore \pi P=\pi$

Where $\pi = (\pi_1, \pi_2)$ and $\pi_1 + \pi_2 = 1$ .... (1)

$$\therefore (\pi_1, \pi_2)\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = (\pi_1, \pi_2) \Rightarrow \begin{bmatrix} \frac{\pi_2}{2} & \pi_1 + \frac{1}{2}\pi_2 \end{bmatrix} = (\pi_1, \pi_2)$$

$$\Rightarrow \frac{\pi_2}{2} = \pi_1 \ \ and \ \ \pi_1 + \frac{1}{2}\pi_2 = \pi_2 \Rightarrow \pi_2 = 2\pi_1 \ \ and \ \ \pi_1 = \frac{1}{2}\pi_2$$

$$(1) \Rightarrow \pi_1 + 2\pi_1 = 1 \Rightarrow \pi_1 = \frac{1}{3} \ \& \ \pi_2 = 2\pi_1 \Rightarrow \pi_2 = \frac{2}{3}$$

$$\therefore \pi = \left(\frac{1}{3}, \frac{2}{3}\right)$$

6. Define TPM and one step TPM.

Solution:

Transition probabilities can be arranged in a matrix form. Such a matrix is called as TPM.

$$P = \begin{bmatrix} P_{11} & P_{12} & .... & P_{1m} \\ P_{21} & P_{22} & .... & P_{2m} \\ .... & .... & .... & .... \\ P_{m1} & P_{m2} & .... & P_{mm} \end{bmatrix}$$

One Step TPM:

The conditional probability $P\left[X_{n+1} = a_j / x_n = a_i\right]$ is called one – step transition probability from state a_i at time t_n to the state a_j at time t_{n+1} in one step.

7. State the four types of a stochastic process.

Solution:

Random process X (t) is classified according to time t and random variable X (t) at time t. Vales of
X (t) are called states of the process.

(i) Discrete time, discrete state RP
(ii) Discrete time, Continuous state RP
(iii) Continuous time, discrete state RP
(iv) Continuous time, continuous state RP

8. State any two properties of a Poisson process.

Solution:

(i) The Poisson process is a Markov process
(ii) Sum of two independent Poisson processes is a Poisson process.

9. Given that X (t) is a random process with mean $\mu(t) = 3$ and autocorrelation function $R(t_1, t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$ Determine the mean, variance and covariance of the random variable $Y = X(5)$ and $Z = X(8)$.

Solution:

Given $\mu_X(\text{t}) = 3 \Rightarrow \text{E}\left[X(\text{t})\right] = 3 \ \forall t$

And $R(t_1, t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$

$$Y = X(5) \ \ and \ \ Z = X(8)$$

$$\therefore \text{E(Y)} = \text{E(X(5))} = 3 \ \ and \ \ \text{E(Z)} = \text{E(X(8))} = 3$$

$$E\left[Y^2\right]=E\left[X^2(5)\right]=R(5,5)=9+4e^{-0.2(0)}=9+4=13$$
$$E\left[Z^2\right]=E\left[X^2(8)\right]=R(8,8)=9+4e^{-0.2(0)}=9+4=13$$
$$Var(\text{Y})=E\left[X^2(5)\right]-\left[\text{E}\left[X(5)\right]\right]^2=13-3^2=4$$
$$Var(\text{Z})=E\left[X^2(8)\right]-\left[\text{E}\left[X(8)\right]\right]^2=13-3^2=4$$
$$Cov(\text{Y},\text{Z})=\text{E}\left[YZ\right]-E(\text{Y})\,\text{E}(\text{Z})$$
$$E(\text{YZ})=\text{E(X(5) X(8))}=\text{R}_{XX}(5,8)=9+4e^{-0.2(5-8)}=9+4e^{-0.6}$$
$$\text{Cov}(\text{Y},\text{Z})=9+4e^{-0.6}-3\times3=2.195$$

10. Consider a random process $X(t)=\cos(t+\phi)$, where ϕ is a random variable with density function $f(\phi)=\frac{1}{\pi},\ \frac{-\pi}{2}<x<\frac{\pi}{2}$, check whether the process is stationary or not.

Solution:

If the random process is stationary then E [X (t)]=Constant

Given $X(t)=\cos(t+\phi)$ and $f(\phi)=\frac{1}{\pi},\ \frac{-\pi}{2}<x<\frac{\pi}{2}$

$$\therefore E\left[X(\text{t})\right]=\int_{-\infty}^{\infty}X(\text{t})\,\text{f}(\phi)\,d\phi=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\cos(\text{t}+\phi).\frac{1}{\pi}\,d\phi=\frac{2\cos t}{\pi}\neq\text{constant}$$

Then X (t) is not a stationary process.

11. Find the mean square value of the random process whose autocorrelation is $\frac{A^2}{2}\cos\omega\tau$

Solution:

Given $R_{XX}(\tau)=\frac{A^2}{2}\cos\omega\tau$

Mean Square value is $E\left[X^2(\text{t})\right]=R_{XX}(0)=\frac{A^2}{2}\cos 0=\frac{A^2}{2}$

12. Let $A=\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ be a stochastic matrix. Check whether it is regular.

Solution:

Given $A=\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ is a stochastic matrix.

$$A^2=\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{4} & \frac{3}{4}\end{bmatrix}$$

Since all the entries of A^2 are positive, A is regular.

13. **Give an example of a Markov process.**
Solution:
Three children A, B, C are throwing a ball at each other. A always throws to B, and B always throws to C. But C is as likely to throw to B as to A. The throw pattern is a Markov process, because the child throwing the ball is not influenced by those who had the ball previously.

14. **When is a Markov chain, called homogeneous?**
Solution:
If $P_{ij}(\mathrm{n-1,n}) = \mathrm{P_{ij}(m-1,m)}$ is called the homogeneous Markov Chain.

PART – B

1. Show that the random process $X(t) = A\cos(\omega t+\theta)$ is WSS stationary, where A and ω are constants and
θ is uniformly distributed on the interval $(0, 2\pi)$.
2. The probability distribution of the process $\{X(t)\}$ is given by
$$P\left[X(t)=n\right]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3...\\ \dfrac{at}{1+at} & ,n=0\end{cases}$$
Show that it is not stationary.
3. If $X(t) = Y\cos t + Z\sin t$, where Y and Z are independent random variables, each of which assumes the values -1 and 2 with probabilities $\frac{2}{3}$ and $\frac{1}{3}$ respectively. Prove that $X(t)$ is a WSS process.
4. The TPM of a Markov chain $\{x_n\}$, $n = 1,2,3,...$ having 3 states, 1, 2and 3 is
$$P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$$ and
the initial distribution is $P^{(0)} = \begin{pmatrix}0.7 & 0.2 & 0.1\end{pmatrix}$ Find (i) $P(X_2 = 3, X_1 = 3, X_0 = 2)$ (ii) $P(X_2 = 3)$
(iii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

5. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train
but if he drives one day, then the next day he is just as likely to drive again as he is to travel by

train. Now
suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability the he drives to work in the long run.

6. An engineer analysing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals followed a highly distorted signal with no recognizable signal, whereas 20 out of 23 recognized signals follow recognizable signals with no highly distorted signals between. Give that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted.

7. Show that the random process $X(t) = A\cos t + \sin t$, where λ is a constant, A and B are random variables, is WSS if (i) $E(A) = E(B) = 0$ (ii) $E(A^2) = E(B^2)$ (iii) $E(AB) = 0$.

8. Three boys A, B, C are throwing a ball to each other. A always throw the ball to B and B always throws to C
but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the TPM and
classify the states.

9. Find the mean and variance of the poisson process.

10. Find the auto correlation function of the poisson process.

11. Sum of two independent poisson processes is a poisson process.
12. The inter arrival time of a poisson process with parameter λ is an exponential distribution with mean $\frac{1}{\lambda}$.
13. Prove that the difference of two independent poisson processes is not a poisson process.

14. If the customers arrive in accordance with the poisson process, with mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute, (2) between 1 and 2 minute, (3) less than 4 minute.

15. Given a random variable y with characteristic function $\phi(\omega) = E\left[e^{i\omega y}\right]$ and a random process defined by $X(t) = \cos[\lambda t + y]$ show that X(t) is stationary in the wide sense of $\phi(1) = \phi(2) = 0$.

16. Given that WSS random process $X(t) = 10\cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that the process X(t) is correlation – ergodic.

17. A house wife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However if she buys B or C, the next week she is 3 times as likely to buy A as the other cereal. In the long run how often she buys each of the 3 cereals?

18. If X (t) is a Gaussian process with $\mu = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$, find the probability that (1) $X(10) \le 8$

(2) $\left|X(10)-X(6)\right|\leq 4$.

UNIT IV

CORRELATION AND SPECTRAL DENSITIES

PART - A

1. **A stationary random process has an auto correlation function and is given by $R_{XX}(\tau)=\dfrac{25\tau^2+36}{6.25\tau^2+4}$. Find the mean of X (t).**

Solution:

Given $R_{XX}(\tau)=\dfrac{25\tau^2+36}{6.25\tau^2+4}$, where X (t) is a stationary process.

WKT $\mu_X{}^2=\lim\limits_{\tau\to\infty} R_{XX}(\tau)$

$$=\lim_{\tau\to\infty}\frac{25\tau^2+36}{6.25\tau^2+4}=\lim_{\tau\to\infty}\frac{25+36/\tau^2}{6.25+4/\tau^2}=\frac{25}{6.25}=4$$

$\therefore \mu_X=2$

Again from property of autocorrelation function

$$E\left[X^2(t)\right]=R_{XX}(0)=\frac{36}{4}=9$$

$$\therefore Var\left[X(t)\right]=E\left[X^2(t)\right]-\left\{E\left[X(t)\right]^2\right\}=9-2^2=5$$

2. **Given the autocorrelation function of a stationary ergodic process with no periodic component is $R(\tau)=25+\dfrac{4}{1+6\tau^2}$. Find the mean and variance of the process.**

Solution:

$$\text{Mean } \mu_X=\sqrt{\lim_{\tau\to\infty}R(\tau)}=\sqrt{\lim_{\tau\to\infty}\left(25+\frac{4}{1+6\tau^2}\right)}=\sqrt{25}=5$$

$$E\left[X^2(t)\right]=R_{XX}(0)=25+4=29$$

$$\therefore Var\left[X(t)\right]=E\left[X^2(t)\right]-\left(E\left[X(t)\right]^2\right)=29-5^2=4$$

3. **Define Cross Correlation.**

Solution:

Cross – Correlation of two random processes X (t) and Y(t) is defined as

$$R_{XY}(t_1,t_2)=E\left[X(t_1)Y(t_2)\right]$$

4. **State any two properties of Cross correlation function.**

Solution:

Properties 1. $R_{XY}(-\tau)=R_{YX}(\tau)$

2. $\left|R_{XY}(\tau)\right|\le\sqrt{R_{XX}(0).R_{YY}(0)}$

5. **Prove that** $R_{XY}(\tau) = R_{YX}(-\tau)$

Solution:

$$R_{XY}(\tau) = E\left[X(t)Y(t+\tau)\right] = E\left[Y(t+\tau)\,X(t)\right]$$

$$= E\left[Y(t_1)X(t_1-\tau)\right] \quad \left[put\ t_1 = t+\tau\ then\ t = t_1-\tau\right]$$

$$= E\left[Y(t_1)X(t_1+(-\tau))\right]$$

$$\therefore R_{XY}(\tau) = R_{YX}(-\tau)$$

6. **Find the mean square value of the random process whose autocorrelation is** $\frac{A^2}{2}\cos\omega t$ **.**

Solution:

Given $R(\tau) = \frac{A^2}{2}\cos\omega t$

WKT mean square value = $E\left[X^2(t)\right] = R(0) = \frac{A^2}{2}\cos 0 = \frac{A^2}{2}$

7. **Check whether** $\frac{1}{1+9\tau^2}$ **is a valid autocorrelation function of a random process.**

Solution:

Given $R(\tau) = \frac{1}{1+9\tau^2}$

$$\therefore R(-\tau) = \frac{1}{1+9(-\tau)^2} = \frac{1}{1+9\tau^2} = R(\tau)$$

$\therefore R(\tau)$ is an even function. So it can be the autocorrelation function of a random process.

8. **Check whether** $R_{XX}(\tau) = \tau^3 + \tau^2$ **is a valid autocorrelation function of a random process.**

Solution:

Given $R_{XX}(\tau) = \tau^3 + \tau^2$

$$\therefore R_{XX}(-\tau) = (-\tau)^3 + (-\tau)^2 = -\tau^3 + \tau^2 \neq R_{XX}(\tau)$$

Since $R_{XX}(\tau)$ is not an even function, it is not a valid autocorrelation function.

9. **The auto correlation** $R(\tau) = 16 + \frac{9}{1+6\tau^2}$ **function of a stationary random process is. Find the variance of the process.**

Solution:

Given $R(\tau) = 16 + \frac{9}{1+6\tau^2}$

WKT $\mu_X = \sqrt{\lim_{\tau\to\infty} R_{XX}(\tau)} = \sqrt{\lim_{\tau\to\infty} 16 + \frac{9}{1+6\tau^2}} = \sqrt{16+0} = 4$

$$\Rightarrow E\left[X(t)\right] = 4$$

$$E\left[X^2(t)\right] = R_{XX}(0) = 16 + 9 = 25$$

$$\therefore Variance = E\left[X^2(t)\right] - \left\{E\left[X(t)\right]^2\right\} = 25 - 4^2 = 9$$

10. What is meant by spectral Analysis?

Solution:

Let X (t) be a WSS process with autocorrelation function $R_{XX}(\tau) = E\left[X(t)\,X(t+\tau)\right]$. Then the Fourier transform of $R_{XX}(\tau)$ is called the power density spectrum or spectral density of the process and is denoted by $S_{XX}(\omega)$. Thus $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-i\omega\tau}\, d\tau$

The problem of spectral analysis is the evaluation of this Fourier transform which enables us to study the frequency domain of the random signal.

11. The power spectral density of a random process X (t) is given by $S_{XX}(\omega) = \begin{cases} \pi & if\ |\omega| < 1 \\ 0 & elsewhere \end{cases}$**.**

Find its auto correlation function.

Solution:

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{i\omega\tau}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} \pi\, e^{i\omega\tau} d\omega = \frac{1}{2}\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-1}^{1} = \left[\frac{e^{i\tau} - e^{-i\tau}}{2i\tau}\right] = \frac{1}{\tau}\sin\tau$$

12. Can $\frac{\omega + 4}{\omega^2 + 5}$ **be a valid power density spectrum?**

Solution:

$$S_{XX}(\omega) = \frac{\omega + 4}{\omega^2 + 5}$$

$$\therefore S_{XX}(-\omega) = \frac{-\omega + 4}{(-\omega)^2 + 5} = \frac{-\omega + 4}{\omega^2 + 5} \neq S_{XX}(\omega)$$

$\therefore$ It cannot be the density spectrum of a process.

13. The power spectral density function of a zero mean wide sense stationary process X (t) is given by $S_{XX}(\omega) = \begin{cases} 1 & if\ |\omega| < \omega_0 \\ 0 & elsewhere \end{cases}$**. Find** $R_{XX}(\tau)$**.**

Solution:

Given $S_{XX}(\omega) = \begin{cases} 1 & if\ |\omega| < \omega_0 \\ 0 & elsewhere \end{cases}$

WKT $R_{XX}(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{i\omega\tau}\, d\omega$

$$=\frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} 1\, e^{i\omega\tau} d\omega=\frac{1}{2\pi}\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-\omega_0}^{\omega_0}=\frac{1}{2\pi}\left[\frac{e^{i\tau\omega_0}-e^{-i\tau\omega_0}}{i\tau}\right]$$

$$=\frac{1}{\pi\tau}\left[\frac{e^{i\tau\omega_0}-e^{-i\tau\omega_0}}{2i}\right]=\frac{1}{\pi\tau}\sin\tau\omega_0$$

14. If $R(\tau)=e^{-2\lambda|\tau|}$ is the autocorrelation function of a random process X (t), obtained the spectral density of X (t).

Solution:

$$S_{XX}(\omega)=\int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$=\int_{-\infty}^{\infty} e^{-2\lambda|\tau|}\, e^{-i\omega\tau}\, d\tau$$

$$=\int_{-\infty}^{0} e^{(2\lambda-i\omega)\tau}\, d\tau+\int_{0}^{\infty} e^{-(2\lambda+i\omega)\tau}\, d\tau$$

$$=\left[\frac{e^{(2\lambda-i\omega)\tau}}{2\lambda-i\omega}\right]_{-\infty}^{0}+\left[\frac{e^{-(2\lambda+i\omega)\tau}}{-(2\lambda+i\omega)}\right]_{0}^{\infty}=\frac{1}{2\lambda-i\omega}(1-0)+\frac{1}{-(2\lambda+i\omega)}(0-1)$$

$$=\frac{1}{2\lambda-i\omega}+\frac{1}{2\lambda+i\omega}=\frac{2\lambda+i\omega+2\lambda-i\omega}{(2\lambda-i\omega)(2\lambda+i\omega)}$$

$$=\frac{4\lambda}{4\lambda^2+\omega^2}$$

15. Find the power spectral density of a random signal with autocorrelation function $R(\tau)=e^{-\lambda|\tau|}$

Solution:

$$S_{XX}(\omega)=\int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$=\int_{-\infty}^{\infty} e^{-\lambda|\tau|}\, e^{-i\omega\tau}\, d\tau$$

$$=\int_{-\infty}^{0} e^{(\lambda-i\omega)\tau}\, d\tau+\int_{0}^{\infty} e^{-(\lambda+i\omega)\tau}\, d\tau$$

$$=\left[\frac{e^{(\lambda-i\omega)\tau}}{\lambda-i\omega}\right]_{-\infty}^{0}+\left[\frac{e^{-(\lambda+i\omega)\tau}}{-(\lambda+i\omega)}\right]_{0}^{\infty}=\frac{1}{\lambda-i\omega}(1-0)+\frac{1}{-(\lambda+i\omega)}(0-1)$$

$$= \frac{1}{\lambda - i\omega} + \frac{1}{\lambda + i\omega} = \frac{\lambda + i\omega + \lambda - i\omega}{(\lambda - i\omega)(\lambda + i\omega)}$$

$$= \frac{2\lambda}{\lambda^2 + \omega^2}$$

16. Define Cross spectral density.

Solution:

Let X (t) and Y (t) be two jointly stationary processes with cross correlation function $R_{XX}(\tau)$. Then the cross power spectrum of X (t) and Y (t) is $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-i\omega\tau}\, d\tau$

Also if $S_{XY}(\omega)$ is given then we have

$$R_{XY}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XY}(\omega) e^{i\omega\tau}\, d\omega$$

17. Given an example of cross – spectral density.

Solution:

The cross – spectral density of two processes X (t) and Y (t) is given by

$$S_{XY}(\omega) = \begin{cases} p + jq\omega, \ if \ |\omega| < 1 \\ 0, \qquad otherwise \end{cases}$$

18. If the auto correlation function of a stationary process is $R(\tau) = 36 + \frac{4}{1+3\tau^2}$. Find the mean and variance of the process.

Solution:

$$\text{Mean } \mu_X = \sqrt{\lim_{\tau\to\infty} R(\tau)} = \sqrt{\lim_{\tau\to\infty}\left(36 + \frac{4}{1+3\tau^2}\right)} = \sqrt{36} = 6$$

$$E\left[X^2(t)\right] = R_{XX}(0) = 36 + 4 = 40$$

$$\therefore Var\left[X(t)\right] = E\left[X^2(t)\right] - \left(E\left[X(t)\right]^2\right) = 40 - 6^2 = 4$$

19. Given the power spectral density $S_{XX}(\omega) = \frac{1}{4+\omega^2}$ find the average power of the process.

Solution:

$$\text{Given } S_{XX}(\omega) = \frac{1}{4+\omega^2}$$

$$\text{WKT } R_{XX}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega) e^{i\omega\tau}\, d\omega$$

Average power of the process $= R_{XX}(0)$

$$\therefore R_{XX}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{4+\omega^2}\, d\omega$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{2^2+\omega^2}\,d\omega=\frac{1}{2\pi}\left[\frac{1}{2}\tan-1\frac{\omega}{2}\right]_{-\infty}^{\infty}$$

$$=\frac{1}{4\pi}\left[\tan^{-1}(\infty)-\tan^{-1}(\infty)\right]=\frac{1}{4\pi}\left[2\times\frac{\pi}{2}\right]=\frac{1}{4}$$

20. Find the power spectral density of a random signal with autocorrelation function $R(\tau)=e^{-|\tau|}$

Solution:

$$S_{XX}(\omega)=\int_{-\infty}^{\infty}R_{XX}(\tau)\;e^{-i\omega\tau}\;d\tau$$

$$=\int_{-\infty}^{\infty}e^{-|\tau|}\,e^{-i\omega\tau}\;d\tau$$

$$=2\int_{0}^{\infty}e^{-|\tau|}(\cos\omega\tau-\mathrm{i}\sin\omega\tau)\,d\tau=2\int_{0}^{\infty}e^{-\tau}(\cos\omega\tau-\mathrm{i}\sin\omega\tau)\,d\tau$$

$$=2\left[\frac{e^{-\tau}}{1+\omega^2}\left[-\cos\omega\tau+\omega\sin\omega\tau\right]\right]_{0}^{\infty}=2\left[\frac{1}{1+\omega^2}\right]=\frac{2}{1+\omega^2}$$

21. Find the auto correlation function of a stationary process whose power spectral density is given by $S_{XX}(\omega)=\begin{cases}\omega^2 & for\ |\omega|\le 1\\ 0 & for\ |\omega|>1\end{cases}$

Solution:

$$R_{XX}(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{XX}(\omega)e^{i\omega\tau}\,d\omega=\frac{1}{2\pi}\int_{-1}^{1}\omega^2\left(\cos\omega\tau+i\sin\omega\tau\right)d\omega$$

$$=\frac{1}{\pi}\int_{0}^{1}\omega^2\cos\omega\tau\,d\omega=\frac{1}{\pi}\left[\omega^2\left(\frac{\sin\omega\tau}{\tau}\right)-2\omega\left(\frac{-\cos\omega\tau}{\tau^2}\right)+2\left(\frac{-\sin\omega\tau}{\tau^3}\right)\right]$$

$$R_{XX}(\tau)=\frac{1}{\pi}\left[\frac{\sin\tau}{\tau}+\frac{2\cos\tau}{\tau^2}-\frac{2\sin\tau}{\tau^3}\right]$$

PART – B

1. Consider the random process X (t) defined by $X(t)=U\cos t+V\sin t$ where U and V are independent random variable each of which assumes the values -2 and 1 with probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Show that X (t) is wide sense stationary and not strict sense stationary.

2. Consider two random processes $X(t) = 3\cos(\omega t + \theta)$ and $Y(t) = 2\cos(\omega t + \theta - \pi/2)$ where θ is a random variable uniformly distributed in $(0, 2\pi)$. Prove that $\sqrt{R_{XX}(0).R_{YY}(0)} \geq |R_{XY}(\tau)|$.

3. State and prove Wiener – Khintchine theorem.
4. The cross power spectrum of real random processes $X(t)$ and $Y(t)$ is given by

$$S_{XY}(\omega) = \begin{cases} a + ib\omega, & |\omega| < 1 \\ 0 & ew \end{cases}.$$

Find the cross correlation function.

5. If $X(t) = Y\cos t + Z\sin t$, where λ is a constant, Y and Z are two independent random variables, $E(Y) = E(Z) = 0$, $E(Y^2) = E(Z^2) = \sigma^2$ and is a constant, prove that X (t) is strict sense stationary process
of order 2.

6. Given that a process X (t) has the auto correlation function $R_{XX}(\tau) = Ae^{-\alpha|\tau|}\cos(\omega_0\tau)$ where $A > 0, \alpha > 0$
and ω_0 are constants. Find the power spectrum of X (t).

7. If X (t) and Y(t) are two random processes and $R_{XX}(\tau)$ and $R_{YY}(\tau)$ are their respective auto correlation
functions, then $|R_{XX}(\tau)| \leq \sqrt{R_{XX}(0).R_{YY}(0)}$
8. Find the power spectral density of a WSS process with auto correlation function. $R(\tau) = e^{-\alpha\tau^2}$
9. The auto correlation of the random binary transmission is given by

$$R_{XX}(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| \leq T \\ 0 & ew \end{cases}$$

. Find the power spectrum.

10. Two random process $\{X(t)\}$ and $\{Y(t)\}$ are given by $X(t) = A\cos(\omega t + \theta)$, $Y(t) = A\sin(\omega t + \theta)$ where A and ω are constants and θ is a uniform random variable over 0 to 2π. Find the cross- correlation function.

11. If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases}\frac{b}{a}\left(a-|\omega|\right), |\omega|\le a\\ 0 \qquad\qquad ,|\omega|>a\end{cases}$.

Find the auto correlation function of the process.

12. The power spectrum of a WSS process X (t) is given by $S_{XX}(\omega)=\frac{1}{(1+\omega^2)^2}$ find the auto correlation and average power.

13. Given the power spectral density of a continuous process as $S_{XX}(\omega)=\frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the mean square value of the process.

14. The power spectral density function of a zero mean wide – sense stationary process X (t) is given by $S(\omega)=\begin{cases}1\ ,\ |\omega|<\omega_0\\ 0 \qquad ew\end{cases}$. Find and show also that X (t) and $X\left(t+\frac{\pi}{\omega_0}\right)$ are uncorrelated.

15. The auto correlation function of a wide sense stationary random process is given by $R(\tau)=\alpha^2 e^{-2\lambda|\tau|}$. Determine the power spectral density of the process.

UNIT V

LINEAR SYSTEMS WITH RANDOM INPUTS

PART - A

1. Define a system. When it is called a linear system?

Solution:

A system is a functional relation between input x (t) and output y (t). Which is given by

$y(t)=f\left[x(t)\right], -\infty<t<\infty,$

The system is said to be linear if for any two inputs $x_1(t)$ and $x_2(t)$ and constants a_1, a_2

$f\left[a_1x_1(t)+a_2x_2(t)\right]=a_1 f\left[x_1(t)\right]+a_2 f\left[x_2(t)\right]$

2. Describe a linear system with random inputs.

Solution:

A system $Y(t)=f\left[X(t)\right]$ is said to be linear if

$f\left[a_1X_1(t)+a_2X_2(t)\right]=a_1 f\left[X_1(t)\right]+a_2 f\left[X_2(t)\right]$

3. Given an example of a linear system.

Solution:

Consider the system f with output ‘ t x (t)’ for an input signal x (t).

$i.e., \ y(t) = f\left[x(t)\right] = t\,x(t)$ Then the system is linear.

For any two inputs $x_1(t)$, $x_2(t)$ the outputs are $t\,x_1(t)$ and $t\,x_2(t)$

Now $f\left[a_1x_1(t)+a_2x_2(t)\right] = t\left[a_1x_1(t)\right] + t\left[a_2x_2(t)\right]$

$= a_1t\,x_1(t) + a_2t\,x_2(t)$

$= a_1 f\left[x_1(t)\right] + a_2 f\left[x_2(t)\right]$

$\therefore$ The system is linear.

4. If the system has the impulse response $h(t) = \begin{cases} \dfrac{1}{2c}, & |t| \le c \\ 0, & |t| > c \end{cases}$ **. Write down the relation between the spectrums of input X (t) and output Y (t).**

Solution:

$$S_{YY}(\omega) = |H(\omega)|^2\, S_{XX}(\omega)$$

But $H(\omega) = Fourier\,transform\,of\ h(t)$

$$= \int_{-\infty}^{\infty} h(t)e^{-i\tau\omega}\,d\tau$$

$$= \int_{-c}^{c} \frac{1}{2c} e^{-i\tau\omega}\,d\tau = \frac{1}{2c}\left[\frac{e^{-i\tau\omega}}{-i\omega}\right]_{-c}^{c} = -\frac{1}{2ic\omega}\left[e^{-ic\omega} - e^{ic\omega}\right]$$

$$\Rightarrow H(\omega) = \frac{1}{c\omega}\left[\frac{e^{-ic\omega} - e^{ic\omega}}{2i}\right] = \frac{\sin c\omega}{c\omega}$$

$$\therefore S_{YY}(\omega) = \frac{\sin^2 c\omega}{c^2\omega^2}\, S_{XX}(\omega)$$

5. Suppose the input X (t) to a linear time invariance system is white noise. What is the power spectral density of the output process Y (t) if the system response is given by

$$H(\omega) = \begin{cases} 1, & if\ \omega_1 < |\omega| < \omega_2 \\ 0, & otherwise \end{cases}.$$

Solution:

Given X (t) is white noise N (t)

$$\therefore S_{XX}(\omega) = S_{NN}(\omega) = \frac{N_0}{2},\ -\infty < \omega < \infty$$

$$\therefore S_{YY}(\omega) = 1^2.\frac{N_0}{2}$$

$$= \begin{cases} \dfrac{N_0}{2}, & if\ \omega_1 < |\omega| < \omega_2 \\ 0, & otherwise \end{cases}$$

6. Define band limited white noise.

Solution:

Noise having a **non zero and constant power spectrumover a finite frequency band** and zero every where else is called **band – limited white noise**.

If N (t) is a band – limited white noise, then its power density spectrum is usually defined as

$$S_{NN}(\omega)=\begin{cases}\dfrac{N_0}{2},\ if\ |\omega|\le\omega_B\\ 0,\ otherwise\end{cases}$$

7. Find the auto correlation function of the white noise.

Solution:

The spectral density of white noise is usually defined as

$S_{XX}(\omega)=\dfrac{N_0}{2}$, where N_0 is a positive constant.

$$\therefore R_{XX}(\tau)=F^{-1}\left[S_{XX}(\omega)\right]$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{XX}(\omega)e^{i\tau\omega}\ d\omega$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{N_0}{2}e^{i\tau\omega}\ d\omega$$

$$=\frac{N_0}{2}.\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\tau\omega}\ d\omega$$

$=\dfrac{N_0}{2}\ \delta(\tau)$, where $\delta(\tau)$ unit impulse function.

8. Define White Noise.

Solution:

A sample function n (t) of a WSS noise RP N (t) is called white noise, if the power spectrum of **N (t) is a constant at all frequencies.**

$$i.e).,S_{NN}(\omega)=\frac{N_0}{2}$$

9. State auto correlation function of the white Noise

Solution:

The auto Correlation function of the white noise is $R_{XX}(\tau)=\dfrac{N_0}{2}\delta(\tau)$

Where $\mathrm{N}_0\rightarrow$ a +ive real valued constant &

$$\delta(\tau)=\int_0^{\infty}\cos\omega\tau\,d\omega$$

10. State any two properties of a linear time invariant system.

Solution:

(i) $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$

(ii) $\mathrm{Y(t)} = \int_{-\infty}^{\infty} h(\mathrm{u})\,\mathrm{X(t-u)}\,du$

11. Define time invariant system .

Solution:

A system $Y(t) = f\left[X(t)\right]$ is said to be time invariant if $Y(t+h) = f\left[X(t+h)\right]$

12. Check whether the following system are casual (i) $Y(\mathrm{t}) = \mathrm{X(t)} - \mathrm{X(t-2)}$ (ii) $\mathrm{Y} = \mathrm{X(t^2)}$

Solution:

(i) The given system is causal because the present value of $Y(\mathrm{t})$ depends only on the **present (or) previous values of the input** $\mathrm{X(t)}$ **.**

(ii) The given system is not causal because the present value of $Y(\mathrm{t})$ depends on the **future value of the input** $\mathrm{X(t)}$ **.**

13. What is unit impulse response of a system?

Solution:

$$\phi(\mathrm{a}) = \int_{-\infty}^{\infty} \phi(\mathrm{t})\,\delta(\mathrm{t}-a)\,dt$$

Where $\delta(\mathrm{t}-a)$ is the unit impulse function of a.

14. State the properties of a linear filter.

Solution:

Let $\{X_1(t)\}$ and $\{X_2(t)\}$ be any two processes and a and b be two constants.

If **f is linear filter** then $f\left[a\,X_1(t) + b\,X_2(t)\right] = a\,f\left[X_1(t)\right] + b\,f\left[X_2(t)\right]$

PART – B

1. Assume a random process X (t) is given as input to a system with transfer function $H(\omega)=1$ for $-\omega_0 < \omega < \omega_0$ If the auto correlation function of the input process is $\frac{N_0}{2}\delta(\tau)$, find the auto correlation function of the output process.

2. If the input to a time invariant, stable, linear system is a WSS process, prove that output will also be a WSS process.

3. If $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$ where A is a constant, θ is a random variable with uniform distribution in $(-\pi, \pi)$ and $N(t)$ is a band – limited Gaussian white noise with a power spectral

density $S_{NN}(\omega)=\begin{cases}\frac{N_0}{2}, & |\omega-\omega_0|<\omega_B \\ 0 & ew\end{cases}$. Find the power spectral density of $Y(t)$. Assume that $N(t)$ and θ are independent.

4. Consider a Gaussian white noise of zero mean and power spectral density $\frac{N_0}{2}$ applied to a low pass RC

   filter white transfer function is $H(f)=\frac{1}{1+i2\pi fRC}$. Find the auto correlation function.

5. Consider a system with transfer function $\frac{1}{1+j\omega}$. An input signal with auto correlation function $m\delta(\tau)+\omega^2$ is fed as input to the system. Find the mean and mean square value of the output.

6. Let X (t) be a WSS process which is the input to a linear time invariant system with unit impulse $h(t)$ and

   output $Y(t)$, then prove that $S_{YY}(\omega)=|H(\omega)|^2 S_{XX}(\omega)$ where $H(\omega)$ is Fourier transform of $h(t)$.

7. Let X (t) be the input voltage to a circuit system and Y (t) be the output voltage. If X (t) is a stationary random

   process with mean 0 and autocorrelation function $R_{XX}(\tau)=e^{-\alpha|\tau|}$. Find (i) $E[Y(t)]$ (ii) $S_{XX}(\omega)$ (iii) the spectral density of Y (t) if the power transfer function $H(\omega)=\frac{R}{R+iL\omega}$.

8. The auto correlation function of the poisson increment process is given by

   $R(\tau)=\begin{cases}\lambda^2 & for\ |\tau|>\xi \\ \lambda^2+\frac{\lambda}{\xi}\left(1-\frac{|\tau|}{\xi}\right) & for\ |\tau|\geq\xi\end{cases}$ prove that its spectral density is given by

   $S(\omega)=2\pi\lambda^2\delta(\omega)+\frac{4\lambda\sin\left(\omega\xi/2\right)}{\xi^2\omega^2}$.

9. A system has an impulse response $h(t)=e^{-\beta t}U(t)$, find the power spectral density of the output Y (t) corresponding to the input.

10. If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that

    $S_{NN}(\omega)=\begin{cases}\frac{N_0}{2}, & |\omega-\omega_0|<\omega_B \\ 0, & ow\end{cases}$. Find the auto correlation of $\{N(t)\}$.

11. If X (t) is a WSS process and if $y(t) = \int_{-\infty}^{\infty} h(u).X(t-u)\,du$ then prove that

(i) $R_{XY}(\tau) = R_{XX}(\tau) * h(\tau)\ (or)\ h(\tau) * R_{XX}(\tau)$ (ii) $R_{YX}(\tau) = R_{XX}(\tau) * h(-\tau)$

All the Best